Chapter 6

Convection and the Environment

We first discuss the two classical necessary conditions long thought to be needed for the production of deep convection, conditional instability and a triggering mechanism. We then discuss more recently developed criteria, which attempt to establish both necessary and sufficient conditions for the development of convection.

6.1 Conditional instability

Figure 6.1 shows a schematic of a conditionally unstable sounding expressed in terms of the moist entropy and the saturated moist entropy plotted as a function of height. *Conditional instability* means that finite energy must be expended to lift a parcel to the point where it is positively buoyant. The decrease in both saturated and unsaturated moist entropy up to some level, followed by an increase with height is universally found in conditionally unstable soundings.

The saturated moist entropy is the moist entropy with the vapor mixing ratio replaced by the saturated mixing ratio. The moist entropy is conserved in both moist and dry adiabatic processes, so the moist entropy of a surface parcel lifted through the troposphere follows the vertical trajectory shown in figure 6.1. The saturated moist entropy of the surface parcel initially follows a line of constant potential temperature, which low in the troposphere slants sharply to the left, as shown. However, when the saturated entropy line intersects the moist entropy line, the parcel becomes saturated. This level is called the *lifting condensation level* (LCL). Subsequently the saturated moist

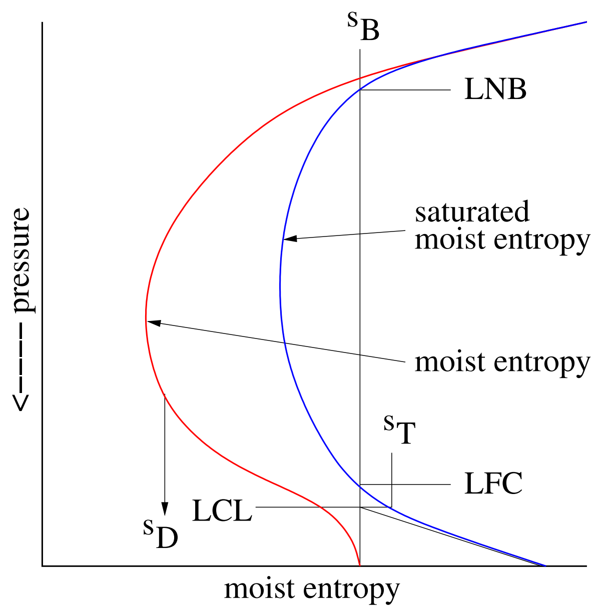

Figure 6.1: Schematic of a typical conditionally unstable atmospheric sounding. LCL is lifting condensation level, LFC is level of free convection, and LNB is level of neutral buoyancy. Plotted are the moist entropy and the saturated moist entropy as a function of pressure.

entropy equals the moist entropy, so both follow the vertical trajectory of the lifted surface parcel.

Neglecting virtual temperature effects, the difference between the saturated moist entropy of a parcel and its surrounding environment is a measure of the buoyancy of the parcel relative to the environment. Thus, when the saturated moist entropy of the parcel exceeds the saturated moist entropy of the environment, the buoyancy of the parcel, which typically is negative initially, becomes positive. This level is called the *level of free convection* (LFC). The parcel will eventually reach its *level of neutral buoyancy* (LNB), after which the buoyancy becomes negative.

The buoyancy of a lifted surface parcel is initially negative relative to the environment (with the exception of a possible shallow region of positive buoyancy over sun-heated land) in almost all cases. Thus, some energy source is required to lift the parcel initially to its level of free convection. The required energy per unit parcel mass is called the *convective inhibition* (CIN). We now derive an equation for CIN. Realizing that the net buoyancy force consists of the downward force of gravity on the parcel plus the upward pressure force which equals the weight of the displaced parcel of environmental air M_e, the work needed to lift the parcel from its initial level ($z = z_I$) to the level of free convection ($z = z_{LFC}$) is the CIN times the mass of the parcel M_p:

$$\text{CIN} \times M_p = \int_{z_I}^{z_{LFC}} g(M_p - M_e)dz, \tag{6.1}$$

where g is the acceleration of gravity. Dividing by M_p results in

$$\text{CIN} = \int_{z_I}^{z_{LFC}} g(1 - \rho_e/\rho_p)dz, \tag{6.2}$$

where ρ_p and ρ_e are the densities of the parcel and the environment.

It is more convenient to express equation (6.2) in the form of a pressure integral, converting from geometrical height to environmental pressure p using the hydrostatic equation $dp = -g\rho_e dz$:

$$\text{CIN} = \int_{p_{LFC}}^{p_I} (1/\rho_e - 1/\rho_p)dp, \tag{6.3}$$

where p_I and p_{LFC} are the pressures at the initial level and the level of free convection.

A convective parcel may contain condensate. To the extent that the hydrometeors have reached terminal fall speed, the mass of this condensate must be included in the calculation of the density of the parcel. If the volume of the condensate in a parcel of volume V is V_c, then the volume of air equals $V_a = V - V_c$. The mass of condensate is $M_c = \rho_w V_c$ where ρ_w is the density of water or ice as appropriate, so the combined density of air and condensate in the parcel is

$$\rho_p = \frac{M_a}{V} + \frac{M_c}{V} = \frac{M_a}{V_a}\left(\frac{V - V_c}{V}\right) + \frac{M_c}{V} = \rho(1 - V_c/V) + \rho_c \qquad (6.4)$$

where ρ is the density of the air and ρ_c is the mass of condensate per unit total volume. The quantity $V_c/V = \rho_c/\rho_w$. Since $\rho_w \approx 10^3$ kg m^{-3} and $\rho_c \approx 10^{-2}$ kg m^{-3} at most, $V_c/V \leq 10^{-5}$ and can be ignored, which means to an excellent approximation that

$$\rho_p = \rho + \rho_c. \qquad (6.5)$$

We use the equation of state for air to compute the parcel density in terms of the pressure p, temperature T, water vapor mixing ratio r_V, and condensate density:

$$\rho_p = \frac{p}{R_D T(1 + 0.61 r_V)} + \rho_c \approx \frac{p}{R_D T(1 + 0.61 r_V - r_C)}. \qquad (6.6)$$

The quantity $\rho_c R_D T/p \approx \rho_c/\rho \approx r_C$ is to good approximation the mixing ratio of condensate, i. e., the ratio of condensate density to dry air density. This series of approximations is valid as long as both r_V and r_C are very much less than unity.

The CIN thus becomes

$$\text{CIN} = R_D \int_{\ln p_{LFC}}^{\ln p_I} [T_e(1 + 0.61 r_{Ve}) - T_p(1 + 0.61 r_{Vp} - r_{Cp})] d \ln p, \qquad (6.7)$$

where a subscripted e indicates environment, a subscripted p indicates parcel values, and where we have used $dp/p = d \ln p$. We have assumed that no condensate exists in the environment. An extended virtual temperature including the effects of condensate is defined

$$T_V = T(1 + 0.61 r_V - r_C), \qquad (6.8)$$

so the integrand is simply the difference between the extended virtual temperatures of the environment and the parcel.

A similar integral yielding the energy released per unit mass in a parcel ascending from the level of free convection to the level of neutral buoyancy is called the *convective available potential energy* (CAPE):

$$\text{CAPE} = R_D \int_{\ln p_{LNB}}^{\ln p_{LFC}} [T_p(1 + 0.61 r_{Vp} - r_{Cp}) - T_e(1 + 0.61 r_{Ve})] d \ln p. \quad (6.9)$$

The order of environmental and parcel quantities is reversed in the integrand compared to the expression for CIN, since CAPE is energy released by the parcel ascent, whereas CIN is the external energy required to lift the parcel.

At a given pressure level the difference between the temperature of the parcel and the environment can be related to the difference between the saturated moist entropies of the parcel and the environment,

$$s_p^* - s_e^* = \frac{\partial s^*}{\partial T}(T_p - T_e), \quad (6.10)$$

where the partial derivative of the saturated moist entropy is taken at constant pressure. (Note that the saturation mixing ratio in the definition of saturated moist entropy is a function of temperature, the effect of which must be included in the partial derivative.) Thus, to the extent that the difference between real and virtual temperatures can be ignored, the CIN can be written

$$\text{CIN} \approx R_D \int_{\ln p_{LFC}}^{\ln p_I} \left(\frac{\partial s^*}{\partial T}\right)^{-1} \left[s_e^*(p) - s_p^*\right] d \ln p. \quad (6.11)$$

Similarly, the CAPE can be approximated

$$\text{CAPE} \approx R_D \int_{\ln p_{LNB}}^{\ln p_{LFC}} \left(\frac{\partial s^*}{\partial T}\right)^{-1} \left[s_p^* - s_e^*(p)\right] d \ln p. \quad (6.12)$$

Above the lifting condensation level the saturated entropy of the parcel may be replaced by the normal moist entropy in these two equations, since parcel is saturated there and the two quantities are identical.

6.2 Control of CAPE and CIN

CAPE and CIN are functions of the characteristics of the convective parcel itself and the environment through which it ascends. Convective parcels are largely produced in the planetary boundary layer, so the characteristics of this layer and the factors controlling it such as surface fluxes and entrainment from the free troposphere are crucial.

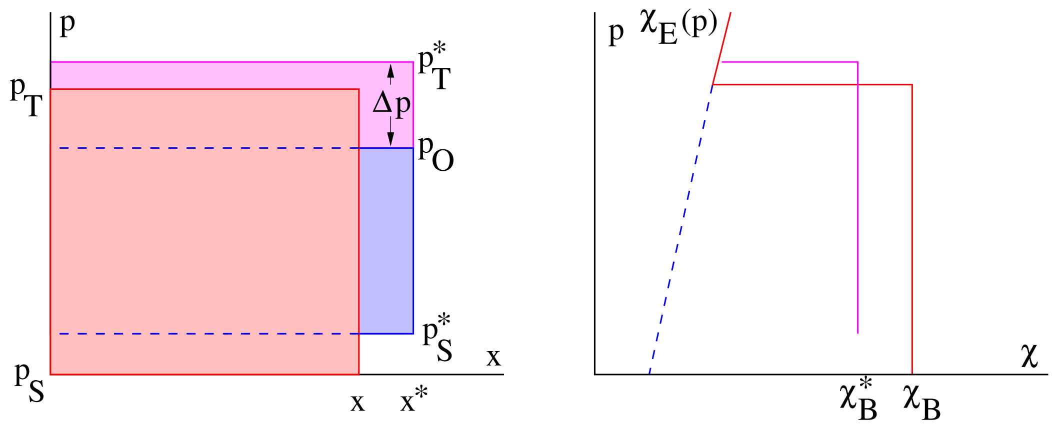

Figure 6.2: The left panel illustrates the evolution of a PBL control volume to stretching or shrinking in the x, y (into page), and p directions plus addition of mass via entrainment from above. The right panel shows the assumed form of any intensive variable χ as a function of pressure p.

6.2.1 Planetary boundary layer

Almost all deep convection originates in a layer in the troposphere called the *planetary boundary layer* (PBL). This is the layer adjacent to the surface in which turbulence acts to mix the layer. Though in practice this mixing is never carried to completion, a useful idealized model of the PBL assumes that the mixing keeps this layer homogenized. Atmospheric motions can distort this layer, increasing or decreasing its thickness and horizontal area. In addition, the PBL generally entrains air from the quiescent free troposphere overlying it.

Figure 6.2 shows how a well-mixed intensive variable χ in a control volume might respond to deformations and entrainment which take place over a time interval Δt, with values at the end of this interval represented by a superscripted asterisk. We assume that the horizontal area of the control volume changes from xy to x^*y^*. Since the pressure is the weight per unit area of the overlying air, the mass in the initial control volume, with base pressure p_S and top pressure p_T is $xy(p_S - p_T)/g$, where g is the acceleration of gravity. With no entrainment, the control volume neither gains nor loses mass. If p_S^* is the base pressure after the elapse of the interval and p_O is the pressure the top would have had at this point in the absence of entrainment, then conservation of mass tells us that

$$xy(p_S - p_T)/g = x^* y^* (p_S^* - p_O)/g. \tag{6.13}$$

Because of entrainment, the actual top of the PBL is $p_T^* = p_O - \Delta p$ at the end of the interval, meaning that the mass in the control volume has increased by approximately $xy\Delta p/g$ during the interval.

We assume that the initial value of χ in the control volume is χ_B and its final value is χ_B^*, while the value in the free troposphere just above the control volume is χ_E. If χ has an internal source per unit mass per unit time of S_χ and a surface flux per unit area per unit time F_χ, then we can compute χ_B^* assuming that it is the mass-weighted average of the pre-existing value in the control volume and the value in the entrained air plus modifications due to the surface flux and the source term:

$$\frac{x^* y^* (p_S^* - p_T^*)\chi_B^*}{g} = \frac{xy(p_S - p_T)(\chi_B + S_\chi \Delta t)}{g} + \frac{xy\Delta p \chi_E}{g} + xyF_\chi \Delta t. \tag{6.14}$$

Letting $\Delta t \to 0$ and invoking equation (6.13) results in the governing equation for χ_B,

$$\frac{d\chi_B}{dt} = \frac{gF_\chi}{p_S - p_T} + \frac{\chi_E - \chi_B}{p_S - p_T}\omega_E + S_\chi, \tag{6.15}$$

where

$$\omega_E = \lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t} \tag{6.16}$$

is the *entrainment velocity*.

Defining the area $A = xy$ and $A^* = x^* y^*$, we note that $p_T^* = p_O - \Delta p$, and that $p_S^* - p_O = (p_S - p_T)A/A^* \approx (p_S - p_T)(1 - \Delta A/A)$, where $\Delta A = A^* - A$. From this we can determine the thickening rate of the PBL in pressure coordinates,

$$\frac{d(p_S - p_T)}{dt} = -(p_S - p_T)\frac{d \ln A}{dt} + \omega_E. \tag{6.17}$$

If we define the horizontal flow velocity in the PBL as $\mathbf{v}_B$, then it is easily demonstrated that

$$\frac{d \ln A}{dt} = \nabla_h \cdot \mathbf{v}_B \tag{6.18}$$

where ∇_h is the two-dimensional horizontal gradient operator.

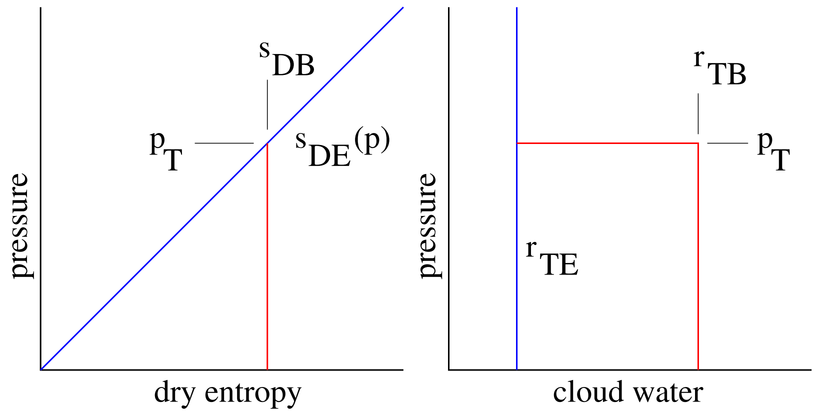

Figure 6.3: Growing cloud-free PBL resulting from surface dry entropy and water vapor fluxes. The dry entropy of the PBL, s_{DB}, matches the dry entropy of the environment, s_{DE}, at PBL top p_T.

We now analyze an example to illustrate how these equations are applied. For this example we assume that $d \ln A/dt = 0$. Figure 6.3 illustrates a cloud-free PBL in an environment with a linear environmental profile of dry entropy $s_{DE} = \Gamma(p_S - p_T)$ (which is conserved in the non-condensing case) and a constant environmental profile of moist entropy s_E. Constant surface fluxes of dry entropy F_d and water vapor F_r are specified and possible source terms (such as radiation for entropy) are ignored. As is observed in developing boundary layers in light winds, the value of dry entropy in the PBL closely matches the environmental value at PBL top. Under these conditions equation (6.15) reduces to

$$\frac{ds_{DB}}{dt} = \frac{gF_d}{p_S - p_T}, \tag{6.19}$$

for the dry entropy and to

$$\frac{ds_B}{dt} = \frac{gF_s}{p_S - p_T} + \frac{s_E - s_B}{p_S - p_T}\omega_E \tag{6.20}$$

for the moist entropy.

The condition on dry entropy at the top of the PBL leads to

$$s_{DB} = \Gamma(p_S - p_T). \tag{6.21}$$

Using this to eliminate $p_S - p_T$ in equation (6.19) leads to a simple differential equation for s_{DB} which has the solution

$$s_{DB} = (2gF_d\Gamma t)^{1/2}. \tag{6.22}$$

Similarly

$$p_S - p_T = \left(\frac{2gF_d t}{\Gamma}\right)^{1/2}. \tag{6.23}$$

Thus, the entropy of the PBL increases with the square root of time, as does the thickness of the PBL. The entrainment velocity equation in this case can be obtained from equations (6.17) and (6.22):

$$\omega_E = \frac{d(p_S - p_T)}{dt} = \left(\frac{gF_d}{2\Gamma t}\right)^{1/2} = \frac{gF_d}{s_{DB}}. \tag{6.24}$$

Using $ds_B/dt = (ds_B/ds_{DB})(ds_{DB}/dt) = (ds_B/ds_{DB})[gF_d/(p_S - p_T)]$ with equation (6.19) and invoking equation (6.24), the differential equation for PBL moist entropy can be simplified to

$$\frac{ds_B}{ds_{DB}} = \frac{F_s}{F_d} - \frac{s_B - s_E}{s_{DB}}. \tag{6.25}$$

This can be rewritten in the form

$$\frac{1}{s_{DB}}\frac{d(s_B - s_E)s_{DB}}{ds_{DB}} = \frac{F_s}{F_d} \tag{6.26}$$

(assuming s_E is constant), which after a bit of algebra, yields the solution

$$s_B = s_E + \frac{F_s s_{DB}}{2F_d}\left(1 - \frac{s_{DB0}^2}{s_{DB}^2}\right) + \frac{(s_{B0} - s_E)s_{DB0}}{s_{DB}} \tag{6.27}$$

where s_{B0} and s_{DB0} are the respective values of s_B and s_{DB} at some specified time. Thus, as the PBL thickens and the dry entropy increases due to surface dry entropy fluxes, the moist entropy in the boundary layer can either increase or decrease with time. A high ratio of surface moist to dry entropy flux promotes an increase with time, whereas a low value of the moist entropy above the PBL promotes a decrease as this air is entrained into the PBL.

6.2.2 Surface fluxes

Scaling arguments originating with G. I. Taylor give us equations for surface fluxes of entropy, moisture, and momentum. For an intensive variable χ, with PBL value χ_B, the surface-air flux is approximately

$$F_\chi = \rho_s C_E(\chi_{ss} - \chi_B)U_{eff}, \tag{6.28}$$

where ρ_s is the air density at the sea surface, $C_E \approx 1 \times 10^{-3}$ is an exchange coefficient, and χ_{ss} is the value of χ for air in immediate contact with the surface. The quantity U_{eff} is the *effective surface wind speed*,

$$U_{eff} = (U_B^2 + W^2)^{1/2}, \tag{6.29}$$

where U_B is the mean PBL wind speed and W is a parameter which accounts for the averaged effect of PBL wind variability, which is important in low wind situations. Typically we find that $W \approx 3$ m s^{-1}. The exchange coefficient C_E is a function of the atmospheric static stability near the surface, with instability leading to larger values of C_E. If strong stability exists in this layer, then C_E becomes small, effectively shutting off the fluxes.

Surface fluxes of three variables are of particular importance, entropy, water vapor, and momentum. Over the ocean, the values of χ_{ss} are well-defined for each of these variables. For entropy and water vapor, χ_{ss} equals respectively the saturated moist entropy and saturation mixing ratio at the temperature and pressure of the sea surface. For momentum, it equals the velocity of the sea surface, generally close to zero. Over land as well as over the ocean, the momentum flux depends on a parameter called the *roughness length* (see Stull, 1988), which has an effect on the value of C_E.

Determination of the entropy and moisture fluxes over land is more difficult than over the ocean. The quantity χ_{ss} cannot be specified as easily as it can over the ocean. Instead, the usual approach is to compute explicitly the budget of heat and moisture in the soil as a function of depth. A byproduct of this approach is the surface fluxes of these quantities. A complicating factor over land is the role of vegetation in transporting moisture from beneath the land surface into the atmosphere. This leads to a large role for biological mechanisms in the surface moisture flux over land.

6.2.3 Geographically forced convection

Geographical inhomogeneities such as topography and variability in surface properties can cause certain regions to be more favorable to the initiation of deep convection. Obvious cases of this convective focusing are the initiation of convection due to the elevated heating of the morning sun on high terrain, and differential solar heating at the land-sea boundary. The net effect in these cases is to modify locally the depth, entropy, and mixing ratio of the PBL, leading to strong local variations in CAPE and CIN. Typically, deep convection forms in regions in which the PBL has become locally thickened, as this thickening has the effect of reducing the convective inhibition. The thickened region of the PBL may also exhibit larger moist entropy, which also reduces CIN and increases CAPE as well.

6.2.4 Large-scale forcing

Geophysical fluid dynamics GFD tells us how the large-scale flow works in the earth's atmosphere. Consideration of this topic is beyond the scope of the course, but the essential point is that GFD exerts a strong control over the wind and temperature profiles in the free troposphere. Thus, the free tropospheric contribution to CAPE and CIN is largely determined by GFD. The interaction between the large-scale flows governed by GFD and the collective behavior of convection is a fascinating topic which we don't have time to discuss here. Large-scale flows also govern (together with convection itself) the distribution of moisture in the atmosphere. The distribution of moisture is of great import to convection.

One aspect of large-scale forcing can be discussed here, namely the effect of large-scale pressure gradients imposed from aloft on the PBL. One of the problems posed below addresses the issue of Ekman balance in the PBL – a situation in which a local balance exists between pressure gradient, Coriolis, and surface drag forces. The actual PBL flow is often not too far from that prescribed by Ekman balance, though recent work suggests that incorporation of the momentum fluxes from air entrained from the free troposphere is necessary for reasonable accuracy in some instances. Convergence in the Ekman balance flow results in thickening of the PBL and possible destabilization to deep convection.

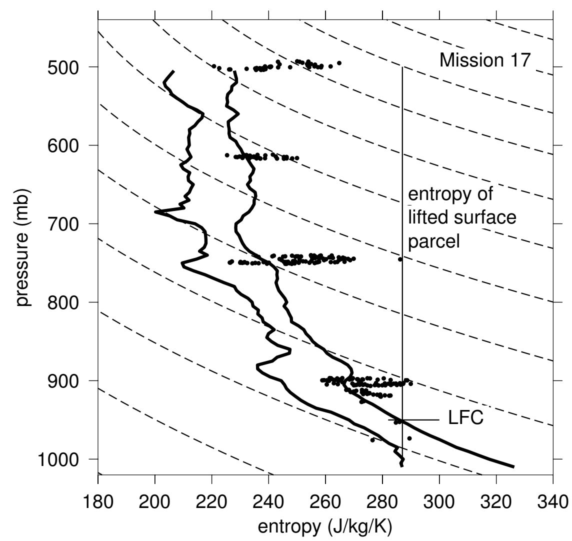

Figure 6.4: Actual sounding and cloud penetrations of a vigorous convective cloud system in the tropical east Pacific (from Raymond et al., 2003). The left solid line is the profile of environmental moist entropy, while the right solid line is the saturated moist entropy of the environment. The dashed lines are lines of constant potential temperature or dry entropy and the dots show moist entropy values inside the clouds. The vertical line shows the moist entropy of a parcel lifted from the lower part of the subcloud layer.

6.3 Development of deep convection

6.3.1 Transition to deep convection

According to the classical picture of convective initiation, convection occurs when sufficient convective available potential energy exists and when the convective inhibition is locally reduced to the point at which a convective parcel can be lifted to the level of free convection by mechanical processes acting in the PBL. Though it is clear that the combination of these two conditions generally produces moist convection, it is not obvious that the resulting convection will be either deep or precipitating, and in most cases it is neither.

The educated eye tells one that deep convection rarely if ever develops directly out of the PBL as conventionally defined, but rather forms from an extensive mass of pre-existing shallow cumulus clouds. These clouds may start out as the well-mixed top of a PBL. However, as they grow, the entrainment of free tropospheric air causes the mean conditions in the cloud layer to deviate significantly from being well-mixed with the underlying sub-cloud layer. The dynamics of this cloud layer is like that of the *mixing layer* envisioned by Frasier (1968) or Raymond and Blyth (1986).

Figure 6.4 shows that values of moist entropy inside a deep convective cloud over the tropical east Pacific do not approach those of the sub-cloud layer, but are characteristic of parcels originating from as high as 800 hPa. This result is typical of measurements in many tropical oceanic clouds as well as those which form over land.

The energy for the mixing in the mixing layer comes from the fact that the moist entropy decreases with height in this layer (see figure 6.1). If the mixing layer is saturated, then the reference profile for static stability is one with constant moist entropy. If the moist entropy decreases with height in this layer, then a vertically displaced moist parcel will acquire a buoyancy which causes it to accelerate away from its initial level. Numerical models of deep moist convection show little evidence of entrainment of environmental air by deep convective thermals at levels above the level of minimum environmental moist entropy.

The mechanism by which deep convective thermals form out of cloudy air in the mixing layer is shrouded in mystery. Such thermals must typically be large enough to ascend to great altitude without experiencing significant entrainment of environmental air. Is the development of such a thermal out of a turbulent field simply the result of a statistical accident? Does the successful thermal gain an advantage by commencing the production of precipitation, leaving less liquid water to cause evaporative cooling upon mixing with environmental air? Does the initiation of ice nucleation lead to additional heating which is crucial to overcoming the CIN? Many such questions remain to be answered.

Evidence is accumulating that deep convection is harder to produce and sustain in environments with low rather than high relative humidity, irrespective of the value of CAPE in the environmental sounding. This is consistent with the idea that evaporative cooling upon mixing with dry air is detrimental to the production of deep convection. However, it is not known whether the mixing in question is of greatest importance in the mixing layer, or whether the dry air saps the thermal primarily after it leaves the mixing layer.

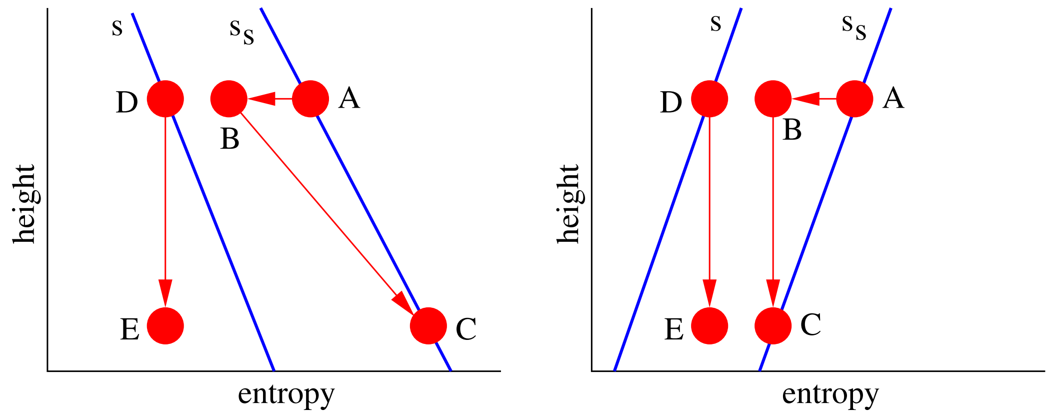

Figure 6.5: Cooling a parcel by evaporation in an environment in which the saturated moist entropy (s_s) decreases (left panel) or increases (right panel) with height. The trajectory segment BC in each case represents the change of saturated moist entropy along a line of constant potential temperature.

6.3.2 Precipitation-induced downdrafts

If precipitation falls out of a cloud into dry air, it will begin to evaporate. This evaporation cools the air, resulting in the formation of a precipitation-induced downdraft. (The weight of the precipitation can by itself cause an air parcel to sink, but this negative buoyancy is reversed as soon as the precipitation falls out of the parcel.) In a layer in which the saturated moist entropy decreases with height, the cooled parcel actually gains capacity to absorb evaporated water as it descends to maintain buoyancy equilibrium with its surroundings.

This phenomenon is shown in figure 6.5. Cooling a parcel at constant height corresponds to moving it to the left in the entropy-height plane, i. e., from point A to point B. The moist entropy of the parcel remains unchanged during this cooling. Since the parcel now has negative buoyancy, it sinks, with the saturated moist entropy following a constant potential temperature line BC. Sinking terminates when the saturated moist entropy of the parcel matches that of the environment. At this level the parcel is again in buoyancy equilibrium with the environment. The moist entropy of the parcel doesn't change in these two processes, which means that the trajectory of the parcel's moist entropy is vertical, DE.

Notice that the difference between moist entropy and saturated moist entropy decreases when the saturated moist entropy increases with height (right panel of figure 6.5), and increases when the saturated moist entropy decreases with height (left panel). The latter generally occurs in the lower part of the free troposphere. An increase in this difference makes precipitation evaporate more readily, whereas a decrease causes evaporation to be slower. Thus, in the lower troposphere where saturated moist entropy decreases with height, sinking due to evaporation of precipitation is a runaway process which accelerates as the parcel sinks in the presence of precipitation, while the reverse is true in the upper troposphere, where the saturated moist entropy increases with height. In the latter case, if the moist entropy of the parcel exceeds the saturated moist entropy of the environment at some level, the parcel cannot sink beyond that level. This implies that downdrafts reaching the surface originate primarily from the level of minimum saturated moist entropy and below.

6.3.3 Convective feedback on PBL

So far we have discussed the possible production of deep convection from the PBL. However, we have not yet discussed the feedbacks imposed on the PBL by deep convection. The most important feedbacks appear to be the extraction of updraft mass from the PBL and its partial replacement by downdraft air. Equation (6.13) representing mass conservation in the PBL becomes

$$xy(p_S - p_T)/g - xy(M_u - M_d)\Delta t = x^* y^* (p_S^* - p_O)/g \tag{6.30}$$

when this is accounted for, where M_u and M_d are the mass per unit area per unit time extracted from and returned to the PBL by deep convection. Similarly, the governing equation (6.15) for an intensive variable χ generalizes to

$$\frac{d\chi_B}{dt} = \frac{gF_\chi}{p_S - p_T} + \frac{g(\chi_d - \chi_B)M_d}{p_S - p_T} + \frac{\chi_E - \chi_B}{p_S - p_T}\omega_E + S_\chi \tag{6.31}$$

where χ_d is the value of χ in convective downdrafts, while equation (6.17) becomes

$$\frac{d(p_S - p_T)}{dt} = -(p_S - p_T)\frac{d\ln A}{dt} - g(M_u - M_d) + \omega_E \tag{6.32}$$

(see Raymond 1995).

The transports of entropy, moisture, and momentum into the PBL by deep convection can have major effects on the PBL. This subject lies at a subdiscipline boundary in the atmospheric sciences, and thus has not received the attention it deserves.

6.4 Precipitation and the environment

Just because there is moist convection does not necessarily mean that rainfall occurs. The convection may occur in an environment which is so dry that entrainment evaporates most of the condensed water. Alternatively, precipitation which does form may fall into a deep, dry boundary layer where it evaporates.

Recent work over the tropical oceans (Bretherton, Peters, and Back 2004) shows a high correlation between the rainfall rate averaged over areas of order a few square degrees and the saturation fraction, defined as

$$\mathcal{F} \equiv \int_0^{p_s} r_V dp \bigg/ \int_0^{p_s} r_S dp \tag{6.33}$$

where p_s is the surface pressure and r_V and r_S are the water vapor mixing ratio and the saturation mixing ratio. The saturation fraction is a kind of column-integrated relative humidity. Alternatively, it may be thought of as the ratio of precipitable water to saturated precipitable water. Figure 6.6 shows an example of this relationship over the eastern tropical Pacific and the southwest Caribbean sea. The applicability of this relationship over land has yet to be determined.

6.5 Form of convection

A vast amount of work has been done in characterizing the morphology of convective systems. This work is well described by Houze (1993) and is treated very briefly here.

6.5.1 Convective and stratiform rain

A typical thunderstorm cell typically produces an initial gush of heavy rain, with the intensity of the rain decreasing gradually over an extended period. Multiple cells in an organized system can produce an extended middle to

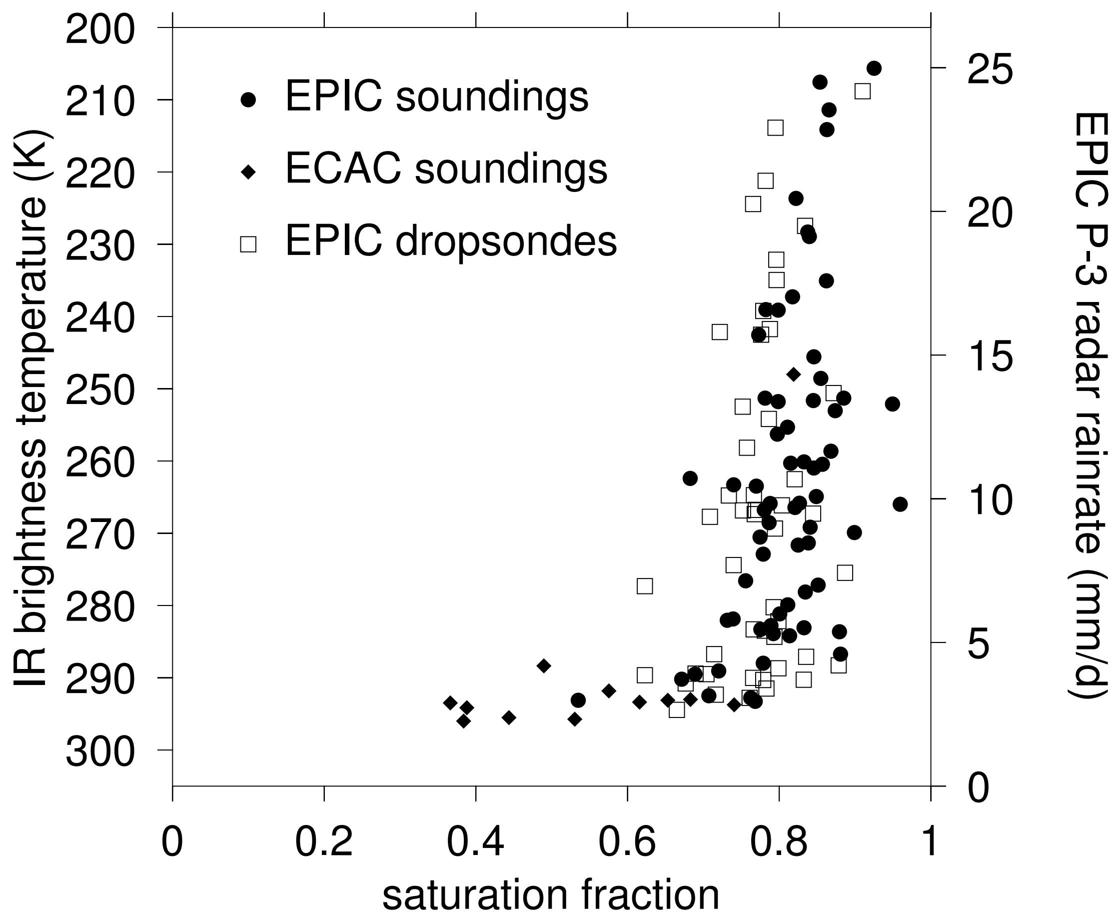

Figure 6.6: Saturation fraction as a function of satellite infrared brightness temperature and inferred precipitation rate in the tropical east Pacific and the southwest Caribbean.

upper tropospheric layer of stratiform cloud which extends the lifetime and amount of this tail of light rain. A formal distinction has been made between *convective rain* (the early period of heavy rain) and *stratiform rain* (the extended tail of light rain). The distinction is an empirical one, but it has some physical basis. The heavy rain is typically the result of heavy riming of ice crystals that fall back through the supercooled water in the cell's updraft. The light stratiform rain generally consists of ice crystals which have been blown out of the top of the updraft into an upper level stratiform layer, where they aggregate into snow flakes. They then fall out of the stratiform cloud, melting at the freezing level and thus turning into small raindrops.

Radar observations of stratiform rain commonly show a *bright band* at the freezing level, i. e., a thin layer of elevated radar reflectivity at this level. This is produced by melting snowflakes which have yet to acquire the higher terminal velocity of raindrops, but have developed higher reflectivity due to the partial melting and development of a surface layer of liquid water. This bright band does not exist in the region of convective rain, because the falling precipitation particles are of higher density and fall speed, and thus melts over a much broader vertical range.

6.5.2 Low shear convection

Low-entropy downdrafts produced by evaporating precipitation spread out in the form of gust fronts when they reach the surface. These gust fronts propagate across the surface, sometimes lifting surrounding PBL air to the level of free convection, thus producing new deep convective cells. If the vertical shear of the horizontal wind is weak, this production of cells and proliferation of gust fronts is somewhat random and disorganized. It basically proceeds until the high-entropy PBL air is exhausted, or at least until the swarm of gust fronts is unable to lift the remaining conditionally unstable air to the level of free convection. Such convection is thus *autocatalytic* to a certain degree, with the initial round of convective cells contributing to the development of others.

6.5.3 Shear and squall lines

When significant wind shear exists in the troposphere, especially in the lowest few kilometers, the lifting of undisturbed PBL air to the level of free convection becomes most efficient on the down-shear side of the downdraft pools of cool air. This can lead to the development of squall lines which propagate downshear, resulting in highly organized, intense convective systems. An extensive literature exists on the morphology and dynamics of squall lines. See in particular the books of Houze (1993) and Emanuel (1994) for more information.

6.5.4 Supercell storms

When the shear is strong enough and the convective available potential energy is large, squall lines give way to a type of self-propagating system called a *supercell storm*. These systems develop cyclonic rotation as the result of incorporating air into updrafts with significant streamwise vorticity. The rotation acts dynamically to cause uplift on one of the flanks of the storm, generally the right flank (facing down-shear) in the northern hemisphere. Houze (1993) describes the propagation mechanism of supercell storms.

These storms are the most violent local storms that occur, and frequently have large hail and strong tornadoes associated with them.

6.6 References

Emanuel, K. A., 1994: *Atmospheric Convection.* Oxford University Press, 580 pp. This is a popular text on convection and has extensive coverage of atmospheric thermodynamics.

Houze, R. A. Jr., 1993: *Cloud dynamics.* Academic Press, San Diego, 570 pp. Robert Houze is the undisputed master of the morphology of convective systems.

Raymond, D. J., 1995: Regulation of moist convection over the west Pacific warm pool. *J. Atmos. Sci.*, **52**, 3945-3959. This paper outlines the theory of "boundary layer quasi-equilibrium".

Stull, R. B., 1988: *An introduction to boundary layer meteorology.* Kluwer, Dordrecht, 666 pp. A reasonably up to date treatment of the subject of boundary layers.

6.7 Problems

1. Using the simplified moist entropy $s = C_{PD}\ln(T/T_F) - R_D\ln(p/p_R) + Lr_V/T_F$ (see the chapter on thermodynamics), compute the factor $\partial s^*/\partial T$ in equation (6.10). (We get s^* from s by replacing the vapor mixing ratio r_V with its saturation value r_S.) Since we are using the simplified form for s in which the latent heat L is constant, this constant value should be used in the differential form of the saturation vapor pressure equation $d\ln e_s/dT = L/(R_V T^2)$.

2. Verify equation (6.18).

3. Suppose a parcel with vapor mixing ratio 18 g kg^{-1} is lifted (without condensing) to an environment where the mixing ratio is 13 g kg^{-1}. At this level the temperatures of the two parcels are both 295 K. How much greater is the virtual temperature of the parcel than that of the environment?

4. Suppose the PBL is supplying deep convective updrafts with mass flux M_u. Suppose further that the convection injects convective downdrafts into the PBL in proportion to the updraft flux: $M_d = \alpha M_u$, where α is a constant. The downdraft air has moist entropy s_d while the updraft is fed by the PBL, with moist entropy $s_B > s_d$.

   (a) Compute the value of $\nabla \cdot \mathbf{v}_B$ needed to keep $p_S - p_T$ constant, assuming that $\omega_E = 0$ and $S_e = 0$.

   (b) If the surface flux of moist entropy F_s and s_B are constant in time as well as $p_S - p_T$, determine M_u. How does this mass flux depend on α and $s_B - s_d$?

5. Ekman balance: Apply the PBL budget equation to the x and y components of the wind in the PBL. For simplicity assume that $\omega_E = 0$ and apply the surface flux equation with $W = 0$ in order to compute surface drag. The source term for the wind contains the horizontal pressure gradient and the Coriolis force per unit mass. For fixed pressure gradient and Coriolis parameter, find the vector horizontal wind in the time-independent case, assuming that $p_S - p_T$ is given.