Physics 222 – Final Exam – Spring 2009

One-page reminder sheet allowed. *Show all work – no credit given if work not shown!* Constants: $\epsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}; $\mu_0 = 4\pi \times 10^{-7}$ N s^2 C^{-2}; $M_{proton} = 938.280$ MeV; $M_{neutron} = 939.573$ MeV; $M_{electron} = 0.511$ MeV; Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J K^{-1}; $\hbar = 8.31 \times 10^{-44}$ BTU fortnight; $c = 1$ ly y^{-1}.

1. According to Dr. Westpfahl's research, stars moving around the center of a galaxy in a circular orbit under the influence of gravity have a constant mean orbital velocity V independent of their distance from the center of the galaxy. The gravitational field is caused by the stars and gas in the galaxy, which we assume to be distributed with spherical symmetry (admittedly, a poor assumption).

   (a) Assuming that stars and gas within a distance R of the galactic center have total mass M, compute V for that orbital radius R. Hint: Gauss's law tells us that this mass acts as if it were concentrated at the center of the galaxy.

   (b) Given the constancy of V, how does M vary with R?

2. Consider an infinitely long, circular cylinder of radius R and mass density ρ.

   (a) Find the gravitational field outside the cylinder as a function of distance r from the cylinder axis.

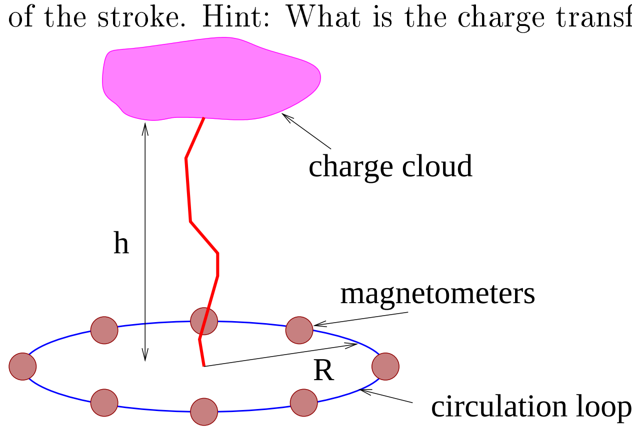
   (b) Find the gravitational field inside the cylinder as a function of r.

3. Lightning hitting the ground has a current of 3×10^4 amp for 5×10^{-5} s. Negative charge is lowered to ground, meaning that the current is upward.

   (a) A circular array of magnetometers $R = 500$ m in radius surrounding the lightning strike zone measures the magnetic circulation around this circle. Compute the magnetic circulation around this loop due to the lightning current.

   (b) If the lightning hits in the center of the array, compute the magnetic field measured by the magnetometers during the stroke.

   (c) If the origin of the charge for the lightning stroke is a compact region of charge $h = 2000$ m directly above the magnetometer array, estimate how much the vertical component of the electric field in the center of the array changes over the period of the stroke. Hint: What is the charge transferred by the stroke?



4. Consider a positively charged particle moving under the influence of scalar and vector potentials $\phi = -Ey$ and $\mathbf{A} = (0, 0, By)$ where E and B are constants.

   (a) Verify that these potentials satisfy the Lorentz condition.

   (b) Compute the electric and magnetic fields.

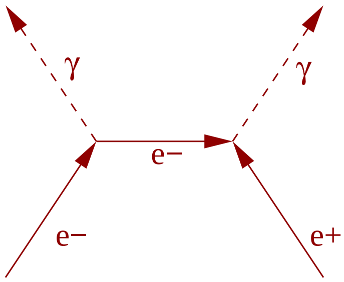
   (c) Determine the velocity (speed and direction) the particle needs to move in order to experience no net force.

5. An electron and a positron with equal energies E_e collide head on and annihilate, resulting in the production of two (non-virtual) photons of equal energy, as illustrated below.

   (a) Compute the momentum of the initial electron.

   (b) Compute the energy and momentum of the left photon.

   (c) Compute the energy and momentum of the virtual electron.



6. A neutron decays.

   (a) What are the decay products?

   (b) How much energy is released?

7. Compute the number of available states $\delta\mathcal{N}$ for a system with entropy $S = 1$ J K^{-1}. Hint: Write $\delta\mathcal{N}$ in the form $\exp(X)$ and calculate X.

8. A real refrigerator operates as shown below.

   (a) How much entropy per unit time is extracted from the interior of the refrigerator?

   (b) How much entropy per unit time is added to the environment?

   (c) How much external power is needed to run the refrigerator?

   (d) How much power would be needed for the same Q_1 if the refrigerator acted as a Carnot engine (running in reverse)?