Design of the New Mexico Liquid Sodium $\alpha\omega$ Dynamo Experiment

1 Design of the Experiment and Feasibility

This is more detailed than would be of general interest, but we were firmly criticized for lack of detailed engineering calculations of the strength of the vessel and the power to drive it to the necessary magnetic Reynolds number. Consequently we calculate the pressure and power as a function of $R_{m,\alpha}$ & $R_{m,\Omega}$ and then use an engineering stress analysis to demonstrate that our constructed experiment meets the criteria of the kinematic dynamo calculations. These criteria are $R_{m,\alpha} = 20$ and $R_{m,\Omega} = 120$ with one pair of plumes per three turns. The size, power and cost of the experiment are are related to these desired values of R_m.

We have chosen cylindrical geometry as the closest approximation to the black hole accretion disk geometry and the most convenient for construction. Spherical geometry would simulate stellar dynamos and make easier the calculation of resistive boundary conditions. We have also chosen Couette flow to minimize the fluid flow dissipation rate and maximize the shear or differential rotation for the Ω flow. For a given desired R_m we will calculate how the pressure and power varies as a function of R_0, namely the choice of a size.

1.1 Centrifugal Pressure

The centrifugal pressure in the fluid sodium determines the required strength of the vessel wall . For Couette flow, $\Omega \propto 1/R^2$ or $v \propto 1/R$ and the fluid pressure becomes:

$$P_{Na} = \rho_{Na}(v_0^2/2)\int_{R_0}^{R_1} \frac{v^2}{R}dR = \rho_{Na}(v_0^2/2)(\frac{R_0^2}{R_1^2} - 1) = (3/2)\rho_{Na}v_0^2 \propto R_{m,\Omega}^2/R_0^2 \qquad (1)$$

when $R_1/R_0 = 1/2$. This ratio is chosen to maximize the annular space, $R_0 - R_1$ for the dynamo and hence maximize $R_{m,\Omega} = (R_0 - R_1)v_0/\eta_{Na}$, yet minimize the acceleration and hence the wall stress at R_1. For rigid-body rotation and the thin-wall approximation, the inner wall stress becomes

$$\tau_1 = a_1\rho_1 = \rho_1 v_1^2/R_1 = \rho_1(v_0^2/R_0)(R_0/R_1)^3. \qquad (2)$$

Therefore for the ratio $R_1/R_0 = 1/2$, the inner wall stress will be $\times 8$ the outer wall stress without fluid pressure. This then allows the outer wall to support primarily the fluid centrifugal pressure rather than its own centrifugal stress, whereas the inner cylinder supports just its own centrifugal stress. Thus for a limiting wall stress or pressure and a desired $R_{m,\Omega}$, the stress decreases as $1/R_0^2$ and so there is a large advantage to size. With this in mind we chose the largest size for which there exists standard bearings, drive belts, mounts, surplus materials, materials handling, and local machine tools for finishing. This size is $R_0 = 30$ cm, and $Z_0 = R_0 = 30$ cm. We next calculate the achievable $R_{m,\Omega}$ and $R_{m,\alpha}$ within the envelope of wall stress and power and compare the results with dynamo calculations.

1.2 Power Requirements

The power required to drive the Ω flow is more complicated to estimate because the process of fluid dissipation is less well known in the particular limits of this experiment. We assume the Couette flow is absolutely stable and so the dissipation within the bulk of the fluid depends only on departures from ideal Couette flow. The primary departure is at the end walls where an Eckman layer forms (Batchelor 1999) of thickness $\Delta Z \simeq Z_0 R_{fliud,\Omega}^{-1/2}$. In our experiment the fluid Reynolds number is $R_{fluid,\Omega} = (R_0 - R_1)v_{Couette}/\mu_{Na} = R_{m,\Omega}(\eta_{Na}/\mu_{Na}) = 9 \times 10^6$, where $R_{m,\Omega} = 120$, $\eta_{Na} = 750$ cm^{2}s^{-1} and $\mu_{Na} = 10^{-2}$ cm^{2}s^{-1}. With $Z_0 = R_0$, the thickness of an Eckman layer becomes $\Delta_{Eckman} = (R_0/2)R_{fliud,\Omega}^{-1/2} \simeq 7 \times 10^{-4}$. The radial flow velocity within the Eckman layers will be $v_{Eckman} \simeq v_\Omega/2$ because the flow is dissipative. Thus the Reynolds number of the flow within the Eckman layer becomes $R_{fluid,Eckman} = R_{fliud,\Omega}^{1/2}/2 \simeq 1.4 \times 10^3$. The flow at this Reynolds number is within the transition to turbulence and so we expect a larger effective viscosity than $\mu_{Na} = 0.01$ and thus a larger thickness, $R_{turbulent,Eckman} > R_{fluid,Eckman}$. Recognizing this uncertainty, the power becomes the kinetic energy dissipated in the fluid flow in a cross-sectional area of $2\pi R \Delta_{Eckman}$. The change in specific kinetic energy occurs between the Couette flow, $v_{Couette}^2/2$ and the wall at $v_0^2(R/R_0)^2/2$. Then the energy loss in two layers becomes

$$\dot{W}_\Omega = \rho_{Na} \int_{R_1}^{R_0} \Delta_{Eckman} 2\pi R (v_\Omega^2 - v_0^2(R/R_0)^2)(v_\Omega/2) dR = \rho_{Na} \pi v_0^3 \Delta_{Eckman} R_0 = 1.5kW. \quad (3)$$

This power is unrealistically small. It implies a spin-down time of

$$\tau_{spindown} = \frac{\rho_{Na} \int_{R_1}^{R_0} 2\pi R (v_\Omega^2/2 - v_0^2/2) Z_0 dR}{\dot{W}_\Omega} = \frac{\rho_{Na} \pi v_0^2 R_0^3 (9/8)}{\dot{W}_\Omega} = \frac{(9/8) R_0^2}{v_0 \Delta_{Eckman}} \quad (4)$$

Thus the spin down time corresponds to a number of turns of

$$N_{turns} = \tau_{spindown} \Omega_0 / 2\pi = (0.86 R_{fluid\Omega_0}^{1/2} / 2\pi \simeq 200. \quad (5)$$

The power predicted is unrealistically small and the number of turns for spin down is similarly unrealistically large. The Eckman flow will be broken up in a complicated fashion by the jet ports in one of the end plates, and so a more realistic estimate of the power required is times ten larger, or 12 kW, which in turn is modest. The scaling with $R_{m.\Omega}$ of Eq. (3) becomes power $\dot{W}_\Omega \propto R_{m,\Omega}^3 / R_0$ so that again for minimizing power there is a significant advantage in large size.

Similarly the plume power is small. The area of two plumes is $A_{plume} = 2\pi r_{plume,0}^2 = 2\pi R_0^2 (r_{plume,0}^2 / R_0^2)$, and so the plume power becomes

$$\dot{W}_\alpha = A_{plume}(v_{plume}^2/2) L_{plume}(\Omega_0/2\pi) n_{plumes/turn} = 2\pi R_0^3 (r_{plume,0}^2/R_0^2)(v_{plume}^2/2)(\Omega_0/2\pi) n_{plumes/turn}. \quad (6)$$

For $v_{plume} = v_0/2$, $n_{plumes/turn} = 1/3$, and $r_{plume,0}/R_0 = 1/6$, we have $\dot{W}_\alpha = 20$ kW. This too is modest and well within a feasible design limit of 50 kW. We expect the plumes and the Ω flow to interact adding to the dissipation of the Ω flow, but this should be proportional to the volume ratio, well within the factor of ten additional dissipation allowed for. Thus we don't see power as the limitation, but instead pressure and the strength of the vessel.

2 Engineering

2.1 The Cylindrical Vessel

The actual design parameters are: an outer cylinder of $R_0 = 30.5$ cm (12 in) with a wall thickness of $\Delta R_0 = 3.2$ cm (1.25 in) and test-space length of $Z_0 = 30.5$ cm (28 in). Since $R_{m,\Omega} = 120$ and $\eta_{Na} = 750$ cm^2 s^{-1}, $v_0 = 6 \times 10^3$ cm s^{-1}. Consequently the pressure of the liquid sodium at the wall from Eq. (1) is $P_0 = 54$ atm or $P_0 \simeq 800$ psi.

The thin wall cylindrical vessel under pressure is the simplest stress analysis, $\sigma_{hoop,Al} = P_{Na}(R_0/\Delta R_0) = 8,000$ psi, (Formula (1b), Table (29), p. 448, "Formulas for Stress and Strain", Roark and Young 1975). Similarly there is an additional stress due to the centrifugal acceleration of the cylindrical vessel itself. In this case for aluminum with density 2.7 and again in the thin wall limit $\sigma_{hoop,Al} \simeq \rho_{Al} v_0^2((R_0 + \Delta R_0)/R_0) = 107atm = 1,570$ psi (p. 566, "Formulas for Stress and Strain", Roark and Young 1975). Since this stress is also a hoop stress, it can be directly added to that from the liquid sodium to give a total hoop stress in the aluminum cylinder of 9,570 psi. The aluminum of the cylinder is 5083H3 which at a temperature of 110C has a yield strength of 32 ksi. Thus we feel that the wall stress at the maximum conditions will be 1/3 the yield strength. As an additional safety measure we expect to band the vessel with stainless steel or Kevlar^{TM}, pre-compressing the vessel by 2000 psi at an internal stress of the banding of half the yield strength of the banding.

The axial stress in the cylinder is determined by the integral of the end wall pressure, Eq. (1), the force F_{end} divided by the circumference and the thickness,

$$F_{end} = \int_{R_0}^{R_1} (2\pi R) P_{Na} dR = 2\pi\rho_{Na}(v_0^2/2) \int_{R_0}^{R_1} R(\frac{R_0^2}{R^2} - 1) dR = \pi R_0^2 v_0^2 (\frac{R_0^2}{2R_1^2} - ln(R_1/R_0)) = 1.31\pi R_0^2 v_0^2. \quad (7)$$

For $\rho_{Na} = 1$ and $R_1/R_0 = 1/2$, the axial stress becomes

$$\sigma_{axial,Al} = \frac{F_{end}}{2\pi R_0 \Delta R_0} = 0.65 v_0^2 (R_0/\Delta R_0) \quad (8)$$

For the desired value $v_0 = 6000$ cm/s, the axial stress becomes 220 atm = 3,300 psi. This adds vectorally to the hoop stress a small $\simeq 10\%$ addition. We will initially perform the tests with the thrust end-plate welded and the driven end-plate bolted using 40 grade 316SS 3/4 - 10 × 3 bolts on a 32 cm (12.62 in) bolt circle. The resulting bolt stress becomes 40 ksi, an acceptable value.

2.2 The End Plates

The thrust end-plate is 6061T651 aluminum of thickness $\Delta Z_{end} = 3.2$ cm (1.25 in), 61 cm (24 in) o.d. with an i.d. of 20.3 cm (8 in). The driven end-plate is 5083H3 aluminum of thickness of $\Delta Z_{end} = 3.2$ cm (1.25 in), 70 cm (27.5 in) o.d. with an i.d. of 17.8 cm (7 in).

The maximum centrifugal stress in these annular plates is given by (Formula (6), p. 567, "Formulas for Stress and Strain", Roark and Young 1975) as

$$\sigma_{end,centrif} = \rho_{Al} \frac{3+\nu}{8} v_{0,end}^2 (1 - (R_{hole}/R_{outer})^2) \simeq v_{0,end}^2 = 17.5\ atm = 250\ psi \quad (9)$$

for the conditions $R_{hole}/R_{outer} = 1/4$ and the desired velocity of $v_0 = 6000$ cm/s. Thus the centrifugal stress in the end plates is is negligible.

The bending stress due to the fluid pressure is more significant. We must consider a circular plate of outer radius $R_{end,outer}$, fixed by the cylinder and loaded with the pressure distribution of Eq. (1) from R_0 to just inside R_1 with the inner edge guided. The bolting or welding to the cylinder serves the purpose of "fixing" the outer radius with the inner radius guided by the flanges and no axial confinement by the bearings. Formula (3_f) of Table (24), p. 345 of "Formulas for Stress and Strain", Roark and Young 1975 best fits this circumstance of outer edge fixed and inner edge guided with a linearly increasing pressure distribution extending from R_1 to R_0. This approximation is conservative relative to the quadratic distribution. The actual case is between these limits. The maximum stress in the plate then becomes

$$\sigma_{end,max} = 6P_0(R_0/\Delta Z)^2 K_{M_R} = 600 P_0 K_{M_R} \quad (10)$$

for both the driven and thrust plates where $R_0/\Delta Z_{end} \simeq 10$. Then for $R_0/R_1 = 2$, Formula (3_f) gives $K_{M_{R_0}} = 0.025$ and $K_{M_{R_1}} = 0.011$. The corresponding stresses become $\sigma_{end,R_0} = 15P_0$ and $\sigma_{end,R_1} = 6.6P_0$. At our desired velocity of 6000 cm/s, these become $\sigma_{end,R_0} = 8000$ psi and $\sigma_{end,R_1} = 3500$ psi. Both of these stresses are modest compared to the strength of 6061T651 Al at 110 C of 32 ksi. The deflection of the plates under this load is given by

$$\delta Z_{end} = K_{y,R_1} \frac{\frac{F_{end}}{2\pi R_0} R_0^3}{D} \quad (11)$$

Where the plate constant, $D = E_{Al}\Delta Z_{end}^3/(12(1-\nu^2)) = 1.8 \times 10^6$. Then for $K_{y,R_1} = 7 \times 10^{-4}$, the deflection on axis $\delta Z_{end} = 3 \times 10^{-3}$ in. This is a negligible axial deflection considering the end play in the thrust bearing and the much larger thermal expansion of $\Delta Z_{thermal} = \Delta T L_0 K_{thermal} \simeq 0.04$ cm. We recognize that this analysis is not exact, but plan to hydrostatically test the vessel, measure the deflections, and compare to these estimates. Additional strength can be added to weak points in the design later if necessary, but the major fraction of the physics is accessible at one half the planned maximum velocity and therefore at 1/4 the stress.

2.3 The Bearings, Dynamic Balance, and Seals

The rotating mass of the vessel and shafts is $M_{rotating,R_0} \simeq 300$ kg. We have designed the bearings of nominal size 5 in, to operate for a "normal" lifetime of 10^8 revolutions (4 months at 30 Hz) with a load of $\times 100$ the rotating mass. Since the acceleration at R_0 is $a(R_0) = 10^3$ g, then we can expect the bearings to support an out-of-balance rotating load of 1/10 the static mass at $R = R_0$. This is absurdly large. The mass of the base and mounts is $\simeq \times 5$ the mass of the rotating apparatus or 1500 kg, so that without additional restraint, the static support and mounts will restrain an out-of-balance mass of $\simeq 1.5$ kg at $R = R_0$. This also is an absurdly large balance error. Our electronic sensors weigh $\simeq 100$ g and we expect to balance the rotating mass to $\simeq 10$ g corresponding to an out-of-balance load of 10 kg at $R = R_0$. Automotive crank shafts are routinely balanced to 0.01 g at $R = 5$ cm.

For Couette flow the inner cylinder rotates at $\Omega_1 = 4\Omega_0$ and so the acceleration becomes $a(R_1) = 8 \times 10^3$ g. This larger acceleration requires a more careful balancing, but since the inner cylinder is a single part whose mass $M_{rotating,R_1} \simeq 20$ kg or $\simeq 0.06 M_{rotating,R_0}$, the degree of balance, 0.1 g at $R = R_1$, is also feasible on an automotive crank shaft balance system. With this degree of out-of-balance mass of 10 and 0.1 g respectively and thus with an out-of-balance load of $M_{load} = 10$ kg, we expect a vibration amplitude of $\Delta R = 2 M_{load} g/(M_{mount} \Omega_{0,1}^2) \simeq 2 \times 10^{-4}$ cm. Since this is much smaller than the bearing clearances, we expect the vibration amplitude to be that of the rotating mass itself resulting in an amplitude of $\times 10$ larger or $\Delta R = 2 \times 10^{-3}$ cm. Again this amplitude of vibration is minor.

We next consider the rotating seals. We plan to operate the experiment with a small quantity of mineral oil, $\sim 10\ to\ 100 cm^3$ in contact and floating above the liquid sodium. The density of oil is $\rho_{oil} \simeq 0.96\ \rho_{Na}$ and consequently floats above liquid sodium. We have performed MHD experiments in the past taking advantage of this small density difference (Colgate et al. 1960) and the oil coating avoids the requirement for using an inert gas in the handling of liquid sodium. Thus, in the rotating frame of the the experiment, the oil "floats" to the axis. We expect to adjust the quantity of oil such that the shaft seals are always bathed in oil, not sodium. This ensures that the industrial problem of seals and liquid sodium is avoided.

2.4 Thermal Properties

Solid sodium is an excellent thermal conductor, $\simeq 1/2$ that of copper. Hence, the thermal time constant of the entire mass, heated or cooled from inside the inner cylinder by hot oil is $t_{heat} = R_0^2/D_{thermal} \simeq 600$ s. Once the sodium is hot enough to liquefy, at 100 C, convection is rapid and the experimental mass becomes isothermal For example, the heating of 20 kW of heat or power dissipation is $t_{heat} = (heat\ mass)/power) =$ 2000 s or 1/2 hour. Similarly an experimental run lasting 10 seconds at high power, e.g. 40 kW, will heat the whole apparatus by 1 deg C without cooling, an acceptable small value. Cooling by windage will be comparable.

3 SAFETY

The fear of sodium may have been fueled by youthful indiscretions in the chemistry lab, yet metallic sodium is a major industrial chemical. Industry has used millions of tons of metallic sodium, shipped in thousands of railroad tank cars of greater than 50 tons each. These tank cars are surrounded by an oil jacket for heating. This is accomplished with negligible risk to the public as determined by DOT. The mass of sodium in each tank car is more than several hundred times the mass of sodium used in our experiment. Sodium is also shipped in barrel size containers where the container is non-returnable. Liquid sodium is the lowest density, high conductivity fluid that is biologically benign. There is a vast difference between using sodium at 600 deg C as a fast reactor coolant and using sodium at 110 C as in this experiment. Much of the high end technology of liquid sodium was developed for the high temperature coolant purpose. Those facilities, built on a massive scale and now surplus, have been used for two recent successful sodium dynamo experiments in East Germany and Latvia (Gailitis et al. 2000; Busse et al. 1996; Rädler et al. 1998). These experiments were performed in immense halls for full containment of a liquid sodium equipment failure. We believe that this scale, cost, and the equipment is an unnecessary burden. We expect to test a different dynamo flow field in more modern and modest equipment, developed in the laboratory and easily moved periodically to and from an adjacent, proven testing facility for high-energy materials (whenever testing with liquid sodium). These tests will be performed in a safety test cell and remotely. This facility, the Energetic Material Research and Testing Center (EMRTC) has a 60 year history of tens of thousands of accident free high explosive tests. Sodium is very safe compared to HE; it cannot detonate. Finally one of us (Colgate 1955) has had extensive hands-on use of liquid sodium in laboratory experiments for determining MHD stability for fusion confinement.

The experiment will be performed in compliance with Federal, State, Industry and Institutional safety practices and procedures involving the receipt, storage, handling, disposition and use of sodium metal. Because of length, the *Safety Practices, Procedures and Compliance* are presented in our web-site *http://www.nmt.edu/~dynamo/*.

A removable, electrically heated, safety shield will surround the cylindrical vessel and will be used whenever the apparatus is rotated and filled with either oil or sodium. This will confine any dynamic sodium loss and absorb the kinetic energy and containment of disrupted parts should a mechanical failure occur.

Operating procedures will include: safety training, balancing, hydrostatic pressure testing with oil, pressure deflection checks, calibration of pressure sensors, relief-valve testing and setting, and thermal controls. Because of length, these *Operating Procedures* are also presented in our web-site for the project: *http://www.nmt.edu/~dynamo/*

4 THEORY: ASTROPHYSICAL APPLICATIONS

Along with the proof-of-principle of the $\alpha\omega$ dynamo, the project also seeks to connect the dynamo to the physical universe. Besides the black hole accretion disk dynamo, we see a natural way to produce an astrophysical dynamo during a core collapse supernova. We propose to simulate such a dynamo using an available numerical code.

Present type II supernova theory (Herant et al. 1994) predicts that a robust explosion occurs only above the forming degenerate core because of neutrino-driven large scale convection in the supernova. This large scale convection in a rotating frame is exactly the conditions necessary for the $\alpha\omega$ dynamo. In fact, such a dynamo may be necessary for the following reason: Ultra strong magnetic fields $\sim 10^{12}$ G are a fundamentally accepted part of neutron star/pulsar theory, but their origin and extraordinary uniformity among pulsars is hard to explain. The standard explanation requires an initial field of 10^8 G in the core of the supernova progenitor star for every case, which seems unrealistic.

We believe that a natural way to arrive at the ubiquitous field of $\sim 10^{12}$ G inferred from the $P\dot{P}$ spin-down in radio pulsars is to create a dynamo during the supernova event that naturally leads to a much stronger initial field and subsequent crustal relaxation leads to the upper limiting field of $\sim 3 \times 10^{12}$ G (Flowers & Ruderman 1977).

An additional consequence of the dynamo is to generate field with high-order multipole moments at the neutron star's surface. Such multipoles could facilitate the modeling of emission profiles which don't fit the simple dipole model (Eilek et al. 2000).

The simulations of the supernova core collapse dynamo will use the existing code, described in Pariev et al. 2001. The current version of the code assumes a boundary which is perfectly conducting. A necessary modification of the code, to include non-conducting boundary conditions, requires solution of the external potential field, which is simpler in spherical geometry and will be undertaken as part of the proposed work.

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