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Date of experiment: 01/28/2021
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Vector Addition of Forces

Introduction

In this lab we are studying vectors and vector addition using a force table. We will be varying both the positions and the masses of the forces. Our goal in each case is to achieve static equilibrium, that is, we will make sure that the forces will be balanced and the net force on the table is zero. We will also show these forces may be balanced graphically and mathematically.

Method

For this experiment we use a force table and metal weights. A force table is a circular table that has pulleys attached to the edge. The pulleys' positions may be adjusted, and the angles are marked on the table. The pulleys are attached on one end to a small ring at the center of the force table, and on the other end to hangers than can hold known masses. The masses on the hangar have vertically directed gravitational force that is translated to a horizontal tension force on the ring through the pulley. Figure 1 shows a top-down view of the experiment set-up.

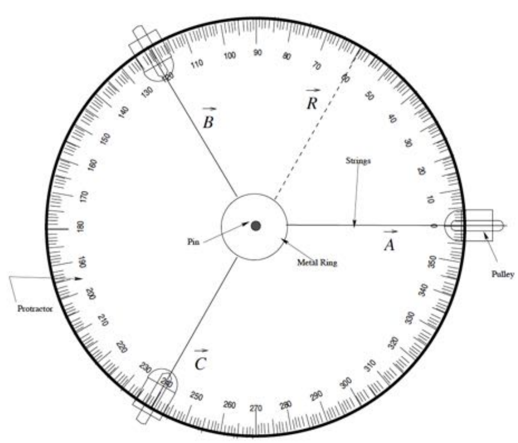


Figure 1: Schematic demonstrating a force table in static equilibrium.
Source: Figure 1.2 from the lab manual

By changing the mass on the mass hangar, we are able to vary the force on the string. We can change the direction of the force by changing the position of the pulley. For this experiment we are presented with two fixed forces and then we can adjust the third to balance the forces. We can see if static equilibrium is met by making sure the central ring is not touching the pin in the center of the force table.

Analysis

To achieve static equilibrium, the net force must equal zero. If we denote the two given vectors as $\vec{A}$ and $\vec{B}$, with vector $\vec{C}$ as being the unknown vector, then

$$\vec{A} + \vec{B} + \vec{C} = 0,$$

And

$$\vec{C} = -(\vec{A} + \vec{B}).$$

We can determine both the magnitude and direction of $\vec{C}$ by carefully drawing vectors $\vec{A}$ and $\vec{B}$ to-scale on graph paper. Vector addition performed graphically requires the vectors to be drawn head-to-tail. For addition, the order of operations does not matter. In this case we can add vectors $\vec{A}$ and $\vec{B}$, and simply multiply the resulting vector by -1, that is, take the opposite direction. An example of the graphical addition of vectors is shown in Figure 2.

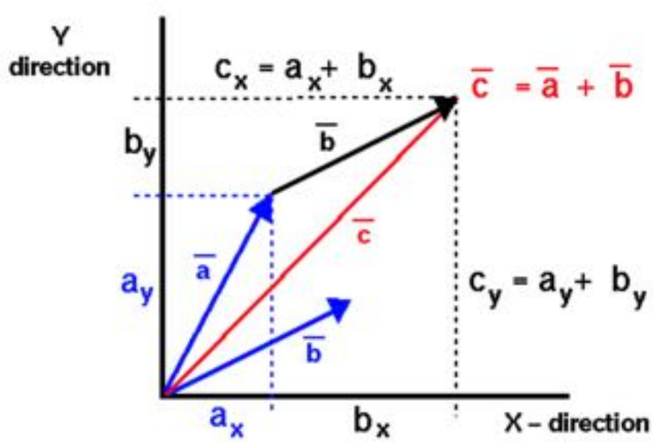


Figure 2: Vector addition performed graphically. Note that in this example $\vec{C} = \vec{A} + \vec{B}$, whereas our experiment requires $\vec{C} = -(\vec{A} + \vec{B})$.
Source: www.grc.nasa.gov

We can also determine the magnitude and direction of vector $\vec{C}$ mathematically by resolving vectors $\vec{A}$ and $\vec{B}$ into their vector components and addition them. For example,

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}\text{,s}$$

And

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath},$$

So

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}.$$

We know that

$$\vec{C} = -(\vec{A} + \vec{B}).$$

So

$$\vec{C} = C_x\hat{\imath} + C_y\hat{\jmath} = -\left(\vec{A} + \vec{B}\right) = -(A_x + B_x)\hat{\imath} - (A_y + B_y)\hat{\jmath},$$

And

$$C_x = -(A_x + B_x)$$

$$C_y - (A_y + B_y).$$

The magnitude of $\vec{C}$ may be determined using the Pythagorean Theorem.

$$\left|\vec{C}\right| = C_x^2 + C_y^2,$$

And the angle may be determined using

$$\theta = \tan^{-1}\frac{C_y}{C_x}.$$

This experiment provides three different cases for us to bring to static equilibrium. Our Instructor suggests that we solve for Case #1 experimentally, and then check it graphically or mathematically, and then for Case #2, solve mathematically and check experimentally.

The magnitudes of the force vectors are determined by the mass of the hanger as well as masses added to the hangers. The mass of the hanger is 50g. The relationship between the force and the mass is $F = mg$, where g is the acceleration due to gravity and is a constant value of 9.8m/s^2.

Table 1 lists the given forces and angles for vectors $\vec{A}$ and $\vec{B}$, as well as the mathematically determined equivalent masses for the forces. The forces, masses, and angles for vector $\vec{C}$ are determined experimentally, graphically and mathematically.

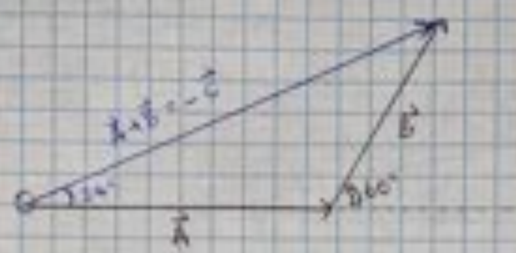
	$\vec{A}$			$\vec{B}$			$\vec{C}$ (experimental)			$\vec{C}$ (graphical)			$\vec{C}$ (mathematical)		
Case #	**Force (N)**	**Mass (g)**	**Angle (°)**	**Force (N)**	**Mass (g)**	**Angle (°)**	**Force (N)**	**Mass (g)**	**Angle (°)**	**Force (N)**	**Mass (g)**	**Angle (°)**	**Force (N)**	**Mass (g)**	**Angle (°)**
1	0.98	100	0	0.686	70	60	1.47	150	203	1.44	146.9	204	1.45	147.99	204.2
2	0.98	100	90	0.98	100	315	0.686	70	204	0.74	75.5	201	0.75	76.54	202.5
3	1.372	140	180	0.98	100	150	2.254	230	349	2.3	234.7	347	2.27	232.05	347.6

Discussion and Conclusion

Overall this experiment was a success. Our graphically determined forces and angles for vector $\vec{C}$ in all three cases was similar to our mathematically determined values. This was also true for our experimentally determined values. The discrepancies in the graphically determined values are likely due to measurement errors when using the ruler and protractor. The discrepancies in the experimentally determined values may be attributed to experimental error such as friction in the pulleys, or poorly levelled force tables. There may also have been errors in reading the angles as the force tables are quite high, and my lab partner (Oliver Boliver Pie) is quite short.

Scale 1grid = 0.1 N = 0.5cm = 5mm

CASE #1



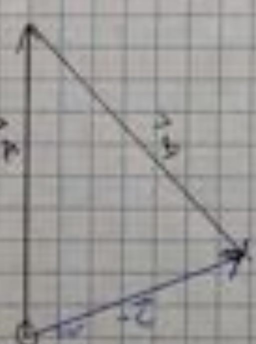
$|C| = 72mm = 1.44\ N$

$\theta = 180 + 24 = 204°$

$\vec{C}$ is in 3^{rd} quad

CASE #2

~~$|C| = 37mm = 0.74N$~~



$|C| = 37mm = 0.74\ N$

$\theta = 180 + 21 = 201°$ (3^{rd} quad [illegible] #7

$2\text{-}3$

$|c| = 11.5\text{ cm} = 115\text{ mm} = $ [illegible] N

$\theta = 360 - 13 = 347^\circ$

(4^{th} quad)