

# Physics 1320 Laboratory Manual

Department of Physics  
New Mexico Institute of Mining and Technology

Spring 2023

## Preface

Our freshman laboratory program aspires to provide students with engaging and rewarding interactions with the physics of the world around them. The weekly laboratories address a cross-section of the concepts introduced in the lecture part of the course. The *Physics Laboratory Manual* is the handbook for the class and introduces the student to the techniques of obtaining and analyzing experimental observations using diverse methods and equipment. The sessions are led by a Teaching Assistant (TA), who is usually a physics graduate student.

Your TA will provide you with a syllabus during the first lab session that will outline the requirements for each lab and its associated laboratory report. In general, students are asked to report their activities and results clearly and concisely in formats ranging from “worksheets” to a formal lab report write-up. The time spent outside of lab on preparation, data analysis, and presentation will vary somewhat on the lab and on the TA, and especially on how much work is done while in the lab. An average time of no more than 3 hours of outside work per lab is encouraged.

Some guidelines to remember throughout the semester:

1. Make sure to always get your raw data sheet signed and/or initialed by your TA prior to leaving the lab session.
2. Punctuality in attendance and completion of assignments.
3. Rotation of individual roles in a team effort to ensure everyone’s participation in the exercises.
4. Approaching all issues with professional courtesy and respecting the efforts, opinions, and property of others.
5. **Disruptive behavior**, including horseplay and reckless use of equipment, will NOT be tolerated!

In an effort to provide the most interesting and interactive physics labs possible, the department of physics at New Mexico Institute of Mining and Technology undertook a significant revision of its lab course manual (Physics 122) during the 2017 – 2018 academic year. This involved major modifications and improvements to already existing labs, and building labs combining ideas from several different sources and universities. Some of the worksheets used for labs are adapted from those written by graduate student Virginie Montes.

Of course, nothing is ever done in isolation, and our efforts relied heavily on the work of those who came before, most notably the original author of a major portion of the labs found within this manual. Also, Dr. Paul Krehbiel and Dr. Barry Sabol provided useful, additional material and information for the labs over the years, which directly led to several of the modifications made. We'd like to also thank current and previous NM Tech physics graduate student instructors and physics professors for their efforts on behalf of this course, including informative and critical feedback.

Carlos Lopez Carrillo  
Keith McElroy

*August 2017*

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# 1 Wave Superposition

In this Lab exercise, we investigate standing waves on a string under tension to introduce the concept of superposition. Superposition is at the base of the study of electromagnetic phenomena and will be investigated in that context in a later lab.

## 1.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

Look up and read about superposition of waves in your physics text.

## 1.2 Introduction

The superposition of oppositely moving waves causes standing waves. In the specific case of a string under tension, the velocity of a traveling wave is given by

$$v_p = \sqrt{F/\mu} \quad (1)$$

where  $F$  is the string tension in Newtons (N) and  $\mu$  is the mass per unit length of the string in  $\text{kgm}^{-1}$ .

In a short high-tension string, like a guitar string, traveling waves are reflected at the endpoint. There are many such reflections for each pluck of the string, leading to the superposition of forward and backward traveling waves.

It can be shown that the superposition of waves (with amplitude  $A$ , angular frequency  $\omega$ , and wave number  $k$ ) that travel in opposite directions can be represented as

$$S(x, t) = 2A \sin(kx) \sin(\omega t). \quad (2)$$

For a given value of  $x$ , say,  $x = x_o$ , the  $\sin(\omega t)$  factor indicates that this ‘point’ is undergoing simple harmonic motion with a amplitude of  $2A \sin(kx_o)$ . For certain values of  $x$ , the amplitude is zero. These points are called nodes. They are found by setting  $\sin(kx) = 0$  which implies that

$$kx = m\pi \implies x = \frac{j\pi}{k}, \quad (3)$$

where  $j$  is an integer, note that the first node occurs for  $j = 0$ , the second for  $j = 1$ , etc.

For this experiment, we would like an expression for the wavelength that can be calculated easily from quantities that are easy to measure on a string exhibiting standing waves. We now derive such an expression.

The definition of wavelength,  $\lambda$  tells us that the distance between nodes denoted  $\Delta x$ , is half the wavelength, that is

$$\lambda = 2\Delta x. \quad (4)$$

Does this make sense based on Figure 1?

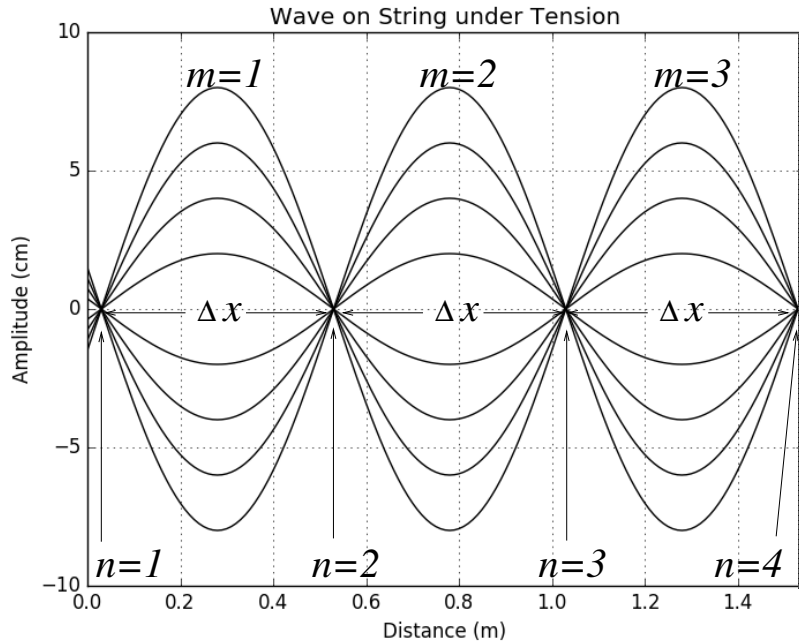


Figure 1: Standing waves on a string with 4 nodes and 3 antinodes. This is sometimes referred to as the third harmonic.

From Figure 1, we can see that the length of the string,  $L$  (measured from the first node to the last node) can be written as

$$L = \Delta x(n - 1) \implies \Delta x = L/(n - 1), \quad (5)$$

where  $n$  is the node number –we need at least two nodes for the equation to be valid. Plugging this into Equation 4 gives us

$$\lambda = \frac{2L}{n - 1} = \frac{2L}{m}, \quad m = 1, 2, 3, \dots \quad (6)$$

where  $m$  is best thought of as the number of antinodes on the standing waveform. Antinodes are the points of maximum amplitude along the string.

### 1.3 Analysis of Standing Waves on a String

The points of minimum amplitude on the string of a guitar are remarkably close to true nodal points. In contrast, the low tension string in this lab is highly dissipative and requires a mechanically driven oscillator at a constant frequency to continuously generate standing waves. In this case, except for the node at the end of the string (away from the driver), the nodal points allow a bit of motion for the energy to flow from the driver to the end of the string and back. Nevertheless, their amplitude is small enough (compared to the amplitude at the antinodes) that they can be considered true nodes.

#### Procedure

Suspend a mass of 150g from the string and tune the driver to produce well-defined envelopes on the string. Find the frequencies that produce 1, 2, 3, . . . , 7 antinodes. Repeat for a hanging mass of 300g. Finally, use a piece of string of the same kind as that of the set up to find the density for this type of string.

**Do Not Disassemble the Setup!**

#### Data

Record the length of the string on your setup, the frequencies that produce each number of antinodes, and the number of nodes. Organize your results in a table. Also, record the mass and length of the sample string to determine its density.

#### Analysis

Determine the density of the string; next, for each hanging mass find the tension on the string; afterward, use Equation 1 to find the predicted velocities for each tension.

Use Equation 6 to find the wavelength. From each frequency, obtain the period,  $T$ . Make a plot  $\lambda$  versus  $T$ . Use a ruler to find the best-fit line for your data and determine its slope (observed velocity). Compare the predicted velocity to the observed velocity.



## 2 Coulomb's Law

In this lab, we investigate Coulomb's law using a torsion balance to measure the repulsion force between two charged spheres.

### 2.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

### 2.2 Introduction

In this exercise, We investigate the Force versus distance between two charged spheres as a function of their separation using the configuration depicted in Figure 1.

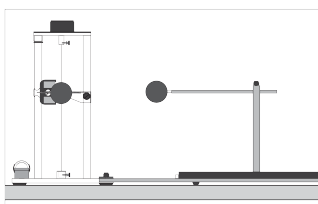


Figure 1: Coulomb's Law Setup.

According to Coulomb's Law, the magnitude of the Force between two point particles charged with charges  $q_1$  and  $q_2$ , respectively, is inversely proportional to the square of their separation.

In general, the formula for Coulomb's law is given by

$$F = k \frac{q_1 q_2}{d^2}, \quad (1)$$

where  $k$  is a constant of proportionality,  $q_1$ ,  $q_2$ , are the excess charge on the particles, and  $d$  their separation.

If the excess Charge on the spheres is unknown, then we could absorb them in the constant of proportionality and rewrite the equation (1) as

$$F = K \frac{1}{d^2}, \quad (2)$$

where  $K = kq_1q_2$  is the new and unknown proportionality constant.

To investigate the relationship between the Force between the charged spheres and their distance, we will use a sensitive torsion balance; the

torsion angle is assumed to be proportional to the electrical Force between the charged spheres. That is

$$F = c\theta, \quad (3)$$

where  $\theta$  is the torsion angle due to the electrical force  $F$ . Therefore, equation (2) can be written as

$$c\theta = K \frac{1}{d^2}, \quad (4)$$

or

$$\theta = C \frac{1}{d^2}, \quad (5)$$

where,  $C = kq_1q_2/c$  is yet another unknown proportionality constant.

Note that for the configuration shown in Figure 1, the particles are not really point-like objects. However, we could assume that they are if their separation is much larger than their diameter. In this case, the conducting spheres behave like point charge particles because the excess Charge resides on the surface and is uniformly distributed over it.

If Coulomb's law, and our assumptions, are satisfied, then equation (5) implies that plotting  $\theta$  against  $1/d^2$  should result in a straight line. We will put equation (5) to the test.

### Procedure

The setup for this experiment is shown in Figure 1. The torsion balance is sensitive enough to detect small amounts of excess Charge on the spheres. However, it is very delicate, so be careful while you work with it. The spheres are non-metallic but covered with graphite to make them conductive. Try not to touch the spheres with your bare hands as you could reduce their conductivity and affect the charge distribution on the sphere.

### Setup

Start by setting the torsion dial to  $0^\circ$ , then rotate the bottom torsion wire retainer until the mark on the vane is aligned with the mark on the index arm, see Figure 2.

Next, position the slide arm at the 3.8 mark of the centimeter scale. Loosen the thumbscrew on the top of the rod supporting the sliding sphere; slide the rod with the sphere until the two spheres just touch and tighten the thumbscrew. Since the sphere's diameter is 3.8 cm, This configuration allows us to read the distance between the centers of the spheres directly off the centimeter scale.

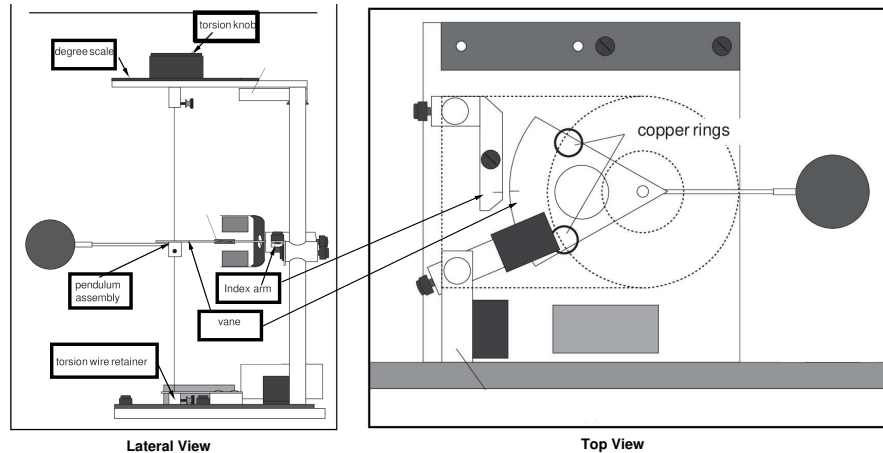


Figure 2: Torsion Balance; lateral and top views.

Make sure that in the current configuration, the degree scale is at zero, the marks on the vane and the index arm are aligned, the spheres are just touching, and the sliding support is at the 3.8 cm mark.

### Charging the spheres

To charge the spheres, use a high voltage but low current power supply. The power supply is grounded and by itself does not pose a risk to your health. Nevertheless, you should not touch the charging probe with your bare hand. Make sure that the negative terminal is also grounded. Ask your instructor if you have questions about the operation of the power supply.

The torsion balance is very sensitive, and many things can affect its performance. In particular, the high voltage at the terminals of the Power supply could affect the measurements. Hence, when charging the spheres, turn on the power supply, charge the spheres and immediately turn the supply off.

Discharge the spheres before charging them by touching them with a grounded probe. Move the sliding support as far away as possible from the pendulum assembly; then, charge the spheres to a potential of 5 kV using the charging probe.

### Measurements

After charging the spheres, position the sliding sphere at the 20 cm mark (the spheres' centers are 20 cm apart according to our setup). Adjust the torsion knob as necessary to realign the marks on the vane and the index

arm. Record the distance between the spheres and the angle,  $\theta$  in a table that includes headings and units. Repeat this measurement until your results are repeatable within two degrees (that is  $\pm 1^\circ$ ).

Once you are satisfied, ask your instructor if you can move on to measure the angles for other distances. Use the set 14, 10, 8, 6 cm, or any set of distances provided by your instructor.

Remember to start each measurement with the spheres discharged and as far as possible from each other and charge them to the same potential.

### Data

Organize the data for each distance in a table that includes headings and units. You should have at least five distances and several rows for each distance.

### Analysis

Start by making a scatter plot of  $\theta$  v.s.  $d$ . Next, determine if your data supports equation (5), by plotting  $\theta$  v.s.  $1/d^2$ . If this graph resembles a straight line, use a ruler to find the best-fit line and determine its slope. The slope is equal to the constant in equation (5).

Determine the value of  $Q$  for each separation distance used, and add the results, including the units, to your data table. You may assume that when you charged the spheres they got the same amount of charge,  $Q$ . Also take the accepted value for the Coulomb's constant in equation (1) as  $k = 8.99 \times 10^9 \text{ Nm}^2\text{C}^{-2}$ , and the value of the proportionality constant in equation (3) as  $c = 1.448574 \times 10^{-6} \text{ N}(\text{degree})^{-1}$ .

Make a scatter plot of the Charge  $Q$  against distance; is this result to be expected? Include the plots in your report as part of the analysis.

## 3 Electric Field and Superposition Principle

This exercise uses a computer simulation to explore further the concept of superposition in the context of electricity. In particular, we study the resultant electric field from various charge configurations.

### 3.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

### 3.2 Introduction

The lab is divided as follows. In section 3.1, the pre-Lab tasks, along with a brief description of the simulation, are given. A brief introduction to the concept of the electric field is offered in section 3.3. The electric field associated with a single charge is explored in section 3.4. The principle of superposition is investigated in section 3.5.

In this computer lab, the simulation found at

<http://phet.colorado.edu/en/simulation/charges-and-fields>

is used to investigate electric fields and the associated principle of superposition. Each student is responsible for getting the simulation working on their own device. It is recommended that you download the simulation and work off-line; Your device should have a diagonal display size over 6".

#### 3.2.1 Working with the Simulation

Before beginning the lab, familiarize yourself with the basic usage of the simulation. Open the PHET simulation in your browser; you should see an image similar to Figure 1.

From the box on the upper-right corner, you can control what is displayed on the screen. Turn off Electric values, and turn on Values and Grid. As seen from the scale at the bottom of the screen, the side of the smallest squares of the grid is 0.1 m; the side for the large squares is 0.5 m. You could drag charges or sensors from the lower bin to any position on the grid. All distances throughout the lab will be given in meters.

*In this lab, we do not explore the full potential of the simulation. You are welcome to explore all aspects of it. Ask your instructor if you have questions about its operation.*

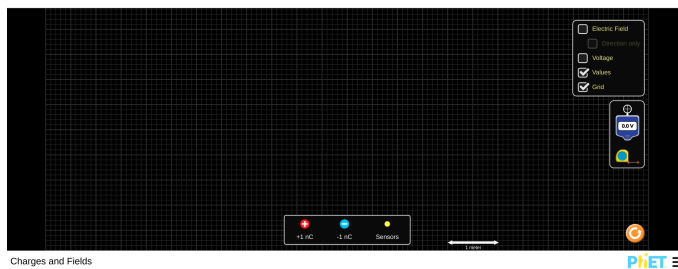


Figure 1: Splash screen from PHET simulation.

### 3.3 Introduction

The electric field is defined in terms of the electric force felt by a small positive test charge located at the point where the field is to be examined:

$$\vec{E} = \frac{\vec{F}_e}{q_0}, \quad (1)$$

where  $\vec{F}_e$  is the electric force acting on the test charge  $q_0$ . Substituting Coulomb's law for the electric force, which is given by

$$\vec{F}_e = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^3} \vec{r}, \quad (2)$$

into (1) we get an expression for the electric field due to a charge  $q$  at a distance  $r$  from that charge:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^3} \vec{r}. \quad (3)$$

In practice, it is easier to calculate the magnitude and direction separately. The direction is found using basic trigonometric techniques, and the magnitude can be found using

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}.$$

Does this equation make sense based on (3)?

**A note about units:** from equation (1), it should be clear that the dimensions of the Electric field are Force per Unit Charge. In consequence, the SI units of the Electric field are Newtons per Coulomb ( $\text{NC}^{-1}$ ). However, it is customary to use dimensions of energy per unit charge for the Electric field; the Volt (V) is defined as one Joule per Coulomb; hence it

is customary to measure the electric field in Volts per meter ( $\text{Vm}^{-1}$ ). The PHET simulation used in this lab displays the magnitude of the electric field in these units.

### 3.4 Single Charge

#### Procedure

After setting up the simulation, drag and drop a 1 nC charge (of either sign) in the middle of the screen. Call this point (0,0) m. Take four yellow ‘Sensors’ out of their bin and place them at (0.5,0), (-1,0), (0,1.5), and (0,-2) m.

#### Data

Use a piece of graph paper to sketch the displayed configuration. On this sketch record the magnitude and direction of the electric field at the location of each sensor in  $\text{Vm}^{-1}$

#### Analysis

Calculate what the electric field should have been at the location of the sensors. Check the simulation results against the theory.

### 3.5 Understanding the Superposition Principle

When more than two point-charges are present, the electric force between each pair of charges is independent of the rest of the charges –each pair acts as if the rest were not there. This observation implies that the net electric force on a test charge, due to a group of charges, is the sum of the forces applied to it by each charge in the group. More simply: find the force on the test charge from each charge in the group, then add the forces, and you will have the total force on the test charge. This way of finding the whole by simply adding the parts is called the superposition principle. The application of this principle to calculate the the electric field produced by various charge configurations is explored in this part of the lab.

#### 3.5.1 Dipole

A Dipole consists of two equal charges of opposite sign separated by a distance,  $d$ . We are going to measure the Dipole’s field at various locations

and compare it with the superposition of the fields produced by the individual charges at the same location.

### Procedure

Take a 1 nC charge and put it in the middle of the screen at (0,0.5) m. Take a  $-1$  nC charge and put it at (0,-0.5) m. We define the center of the Dipole to be the point (0,0) m.

Place a sensor at (0,1) or (0,2) or (0,3) or (0,4) m or any other location specified by your instructor. Use another piece of graph paper to sketch the displayed configuration; label this sketch as 'Dipole.' On this sketch, record the magnitude and direction of the electric field at the location of the sensor in  $\text{Vm}^{-1}$

Leave the sensor in its place, but return the negative charge to its bin. Use the Dipole's sketch to record the electric field's value and direction produced by the positive charge at the sensors' location. Ensure that this electric field is clearly distinguished from the electric field produced by the dipole configuration.

Next, return the negative charge to its original position in the Dipole and remove the positive charge. On the Dipole's sketch, record the electric field's value produced by the negative charge at the position of the sensors. Ensure that this field is clearly distinguished from the previous cases.

### Data

Make a table to record the values and directions of the electric fields obtained in this exercise; on the first two columns, record the value and direction produced by the Dipole; on the next two, the value and direction of the field produced only by the positive charge; use the last two columns for the field produced only by the negative charge. The table's columns should have headers that include a descriptive name and units.

### Analysis

Use the superposition principle to combine the results from the positive and negative, and compare your results with those obtained when both charges were present.

#### 3.5.2 quadrupole

A quadrupole basically consists of two Dipoles oriented in opposite directions. We can think of it as four charges arranged in a square. We are

going to measure the field at the center of the quadrupole and compare with the superposition of the fields produced by the individual charges at the same location.

### Procedure

Select four charges and arrange them in a square of side 2 meters to form a quadrupole. Place a field sensor at the center of the square, and measure the electric field there. Keep the electric field sensor in its place, but return all charges to their bin, except for the one on the upper left corner of the square. Record the electric field due to this charge. Return this charge to its bin and bring back the charge at the upper right corner; record the electric field due to this charge. Do the same for the charges at the other corners.

### Data

Sketch of the charge configuration of your quadrupole. Make a table with columns with headers with units to record the electric fields' value at the center of the quadrupole. Use the first column to record the field due to the quadrupole.

### Analysis

Superimpose the results from the individual charges and compare to the result when the four charges were present simultaneously.

## 3.6 Questions

1. Considering the results from section 3.5.2, what would be the electric field at the center of the square if the charge at the upper-left corner is removed?  
Do not use the simulation to produce the answer; show your calculations.
2. Use equation (1) to verify the equivalence between Newtons per Coulomb and Volts per meter.



## 4 Electric Field Mapping

This Lab exercise uses the electric equipotential lines to map the electric field produced by an electrode configuration.

### 4.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

### 4.2 Introduction

The purpose of this lab is to map the electric field for various electrode configurations. In contrast with the previous computational laboratory, in which you were given the electric field vector, you must experimentally determine the electric field in this lab.

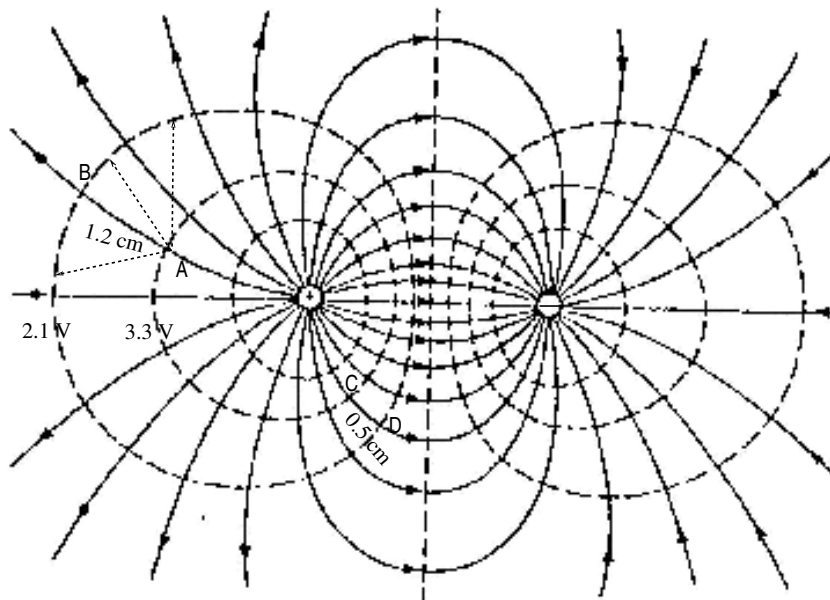


Figure 1: Contours of constant electric potential (equipotentials) and lines of the Electric Field between two point charges of equal magnitude and opposite polarity. The electric field lines are shown as solid lines pointing from high potential to low potential (indicated by arrows). The equipotentials are shown as dashed lines. The two sets of lines always intersect at right angles.

One approach for mapping the electric field vector is to first map the electric potential, which is a scalar field. Mapping the electric potential

means finding regions of equal potential, called equipotentials, for the area surrounding the electrodes. For the cases considered in this lab, the equipotentials are simple lines. Figure 1 shows an example of an equipotential map for a dipole configuration. Note that only a selected group of equipotential lines are shown; in principle, we could find the equipotentials for any value of the electric potential.

We show in the next section that the direction of the electric field must be perpendicular to the equipotentials, that is, it must point in the direction of decreasing electric potential, and that its magnitude is related to the changes in electric potential by Equation 3, below.

An advantage of the equipotentials' method is that for many situations the electric potential difference required in Equation 3 is readily measured with a DMM (Digital MultiMeter). The setup in this lab allows us to use a DMM to map equipotential lines. The potential difference between two points is referred as the voltage between the points, and it is what a DMM can measure. Because of this, we need a reference point to assign an electric potential value to each equipotential. Conventionally, the reference point is given the value of zero, and it is called ground (this convention is similar to the one used to measure gravitational potential from the surface of the Earth).

To illustrate the use of equipotential maps, let us examine the values in Figure 1. Since the lines are equipotentials, placing the probes of the DMM on any two points on the same line should result in a reading of zero volts. On the other hand, putting the positive probe on the line labeled 3.3 V and the negative probe on the one labeled 2.1 V should give a reading of 1.2 V. You should notice that the separation between these two particular equipotentials is not constant. Furthermore, starting at one point over one of these lines, the other could be reached in many ways (see the dashed lines emerging from point A). However, there is one path that is perpendicular to both equipotentials; the distance along this path is 1.2 cm. Hence, using Equation 3, the magnitude of the average electric field, between points A and B, is just 1.0 V/cm; the direction goes from the high (3.3 V) to lower the (2.1 V) equipotential (shown by the solid arrow emerging from point A). For the electric field emanating from point C, the voltage between the equipotentials is the same, but the distance is smaller (0.5 cm). Hence, the magnitude of the average electric field that goes from C to D is (2.4 V/cm). From this example, it should be clear that for areas where the equipotentials are tightly packed, the electric field is more intense, because the changes in electric potential occur over smaller distances.

For the student familiar with topographical maps, it is worth pointing out that contours of equipotentials lines are analogous to contours of elevations on topographical maps, where the lines indicate paths with the same elevation. Similar to equipotential maps, the proximity of elevation

lines in topographical maps indicates regions with large drops in elevation over short distances. In this analogy, positive charge is like water running downhill; electric charge moves from higher to lower electric potential and flows faster through regions with stronger variations on the equipotential map. The setup for this lab places the positive electrode at the top of a ‘hill’ and charge will flow ‘downhill’ towards the negative electrode.

### 4.3 $\vec{E}(V)$ Relationship

To understand the theory behind the retrieval of the electric field from equipotential maps, we now examine the changes in electric potential for a charged particle in an electric field.

By definition, the electric potential is the electric potential energy per unit charge  $\Delta V = U/q$ . Hence, we begin by considering the changes in potential energy for such particle. In general, but not always, an external force does work in moving the particle between two points. The work done is partially stored as potential energy and partially used to increase the kinetic energy of the particle. In the special case when the particle’s kinetic energy does not change (constant speed), all work done is stored as potential energy of the particle. In this case, the external force must exactly balance the electric force, otherwise the particle would be accelerated increasing the particle’s kinetic energy. Therefore for the equilibrium case, the external force must be equal in magnitude and opposite to the electric force, that is  $\vec{F} = -(q\vec{E})$ . In this case, the work done by the external force can be written in terms of the electric field. Assuming that the particle’s displacement is small enough, the electric field may be considered constant. Hence, by definition, the work is simply

$$W = -(q\vec{E}) \cdot \Delta\vec{d}, \quad (1)$$

where  $\Delta\vec{d}$  is the net displacement vector between the initial and final points of the particle’s path. The dot product in Equation 1 implies that work is either only done by the component of the external force parallel to the displacement vector ( $\Delta\vec{d}$ ) or that it is only done when moving the charge parallel to the full electric field vector. Taking the second alternative, the work can be written as  $W = -q|E|\Delta d_{\parallel}$ , where  $\Delta d_{\parallel}$  is the component of the displacement vector parallel to the electric field vector. Since this work is only used to change the electric Potential energy of the particle, we have that  $\Delta U = -q|E|\Delta d_{\parallel}$ . It follows that the change in electric potential (change in potential energy per unit charge) can be written as  $\Delta V = -|E|\Delta d_{\parallel}$ . Solving for  $|E|$ , results in

$$|E| = -\frac{\Delta V}{\Delta d_{\parallel}}. \quad (2)$$

Note that Equation 2 implicitly requires the direction of the electric field to find the component of the displacement parallel to it ( $\Delta d_{\parallel}$ ).

The direction of the electric field is found indirectly by determining the direction perpendicular to it, which is readily determined by tracing the equipotentials (regions of equal potential). The reason is that by definition there is no change in electric potential along the equipotentials. As a result, the change in potential energy for a particle moving along an equipotential is zero, which implies that an external force in equilibrium with the electric field ( $\vec{F} = -q\vec{E}$ ) cannot do Work to increase the potential energy of a charged particle along an equipotential. Therefore, the dot product of the electric field with a displacement vector parallel to an equipotential must be zero. Because we know that the displacement vector is not zero in general along an equipotential, we conclude that the electric field must be perpendicular to them.

Furthermore, to increase the potential energy of the particle, the external force must be directed from low to high equipotentials, implying that the electric field must be directed from high to low values of the electric potential.

In summary, the electric field vector is perpendicular to the equipotentials at any point, its direction points from higher to lower electric potential and its magnitude is given by

$$|E| = \left( \frac{\Delta V}{\Delta d} \right)_{pp}, \quad (3)$$

where  $\Delta V$  is the electric potential difference between two equipotentials, and  $\Delta d$  is the distance between the equipotentials along a path perpendicular to them. The subscript  $pp$  is there to remind us that the change must be measured along this specific path.

It should be pointed out that the magnitude of the electric field could be defined at a single point by taking the limit when  $\Delta d$  goes to zero, in Equation 3. However, for finite values of  $\Delta d$ , the magnitude of the electric field given by Equation 3 is interpreted as the average value between equipotential lines.

Finally, notice that work and  $U$  have dimensions of energy, while the dimensions of  $\Delta V$  are energy per unit charge; the SI units for  $\Delta V$  are joules per coulomb; this combination of units is called the volt.

#### 4.4 Maps

By applying a potential difference across the electrodes, an electric field is generated between them. A DMM (Digital Multimeter) will be used to measure the potential differences between the electrodes and their surrounding points. This requires the media between the electrodes to have low resistivity, which in our setup is provided by a piece of carbon paper placed in contact with and between the two electrodes.

## Procedure

The equipotentials will be measured for two configurations of electrodes, corresponding to the two-dimensional cross-sections of:

1. two parallel plates
2. a coaxial cable.

The conductors are charged by connecting them to a DC power supply, which provides a constant potential difference across its terminals. The potential difference between the electrodes and their surrounding points is measured using a DMM.

### Two Parallel Plates.

This setup mimics a parallel plate capacitor. Set the DC power supply to 10 V and apply the positive and negative terminals to opposite electrodes. Use the DMM to find a value of 1 V on the carbon paper. Trace the location of the equipotential for 1 V parallel to and between the conductors, outside the conductors, and around the edges. One student in your group should find the location of the equipotential points while another student reproduces the locations on a copy of the grid. While symmetry may exist in theory, the carbon paper is not perfect and there will be defects that will affect your equipotentials. Ensure there are enough data points to show the edge effects of the conductors and the behavior inside the capacitor.

Repeat this for equipotentials of 2 V, 3 V, 4 V, ... until you get to 9 V. Use different symbols to distinguish between points of different potential differences.

### Coaxial Cable.

This setup mimics the electric field inside of a coaxial cable, like a television cable. Set the DC power supply to 5 V and apply the positive lead to the inner electrode and use the negative terminal to ground the outer electrode. Use the DMM to find a value of 0.5 V on the carbon paper. Follow the steps as before and find enough data points to represent the equipotential contour for 0.5 V.

Repeat this for equipotentials of 1 V, 1.5 V, 2 V, ... etc. until you get as close to 4.5 V as possible. Explore the magnitude of the potential difference at locations outside the outer grounded electrode and take some notes or record data about what you find.

## Data

Data for each configuration must be collected on its own copy of the grid and different symbols used to distinguish between points of different potentials. Include a legend to identify your symbols. Make sure that you collected enough data by tracing the equipotential lines. Keep the original in case you need to make more copies. Before you leave the lab, have your instructor initial your original data and the copies.

## Analysis

For each electrode configuration, use a different color or line type to make a sketch of the electric field lines on your copy of the data. Recall that the electric field is perpendicular, at every point, to the equipotential lines and its direction runs from high to low values of electric potential difference. Illustrate its direction using small arrow heads along the electric field lines. Your sketch should resemble Figure 1.

In each sketch, locate the most direct (shortest) electric field line between the two electrodes. Make a plot of the electric potential as a function of distance along this particular path. Next, measure the horizontal distance between each pair of adjacent equipotentials along this path and calculate the magnitude of the electric field using Equation 3. The resultant value may be considered the average electric field between the pair of adjacent equipotentials. Repeat this for each adjacent pair of potentials until you have values over the entire path. Plot this average field as a function of distance along the path.

Make sure to have proper labels, legends and the same horizontal scale for both plots (potential and field magnitude). Include your sketches and plots as part of your analysis.

### 4.5 Questions

1. Describe what you found about the potential difference values outside of the coaxial cable setup. Explain this using Gauss' Law.

## 5 Capacitors and Dielectrics

In this lab, we use an LCR meter to measure the value of the capacitance for various geometries and the effective capacitance for capacitors connected in series and parallel. In addition, the effect of a dielectric material inside the parallel plate capacitor is also explored.

### 5.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

### 5.2 Introduction

The capacitor is a device that can store electric charge. The charge is stored on two metal plates inside the capacitor which are insulated from each other. Since the capacitor is always neutral, the metal plates contain an equal amount of negative and positive charge. One of the metal plates contains the negative charge while the other plate has the positive charge. The metal plates can have many shapes. In this lab we will examine two different plate configurations: parallel flat plates and concentric cylindrical plates.

Various dielectric materials can be placed between the plates of a capacitor. The effect of the dielectric on the behavior of a capacitor is given by a unitless parameter called the dielectric constant,  $\kappa$ . It has a value of 1 for vacuum and is very close to 1 for air.

The capacitance,  $C$ , of a capacitor is defined as the ratio of the charge,  $Q$ , on a surface of the capacitor to the potential difference,  $\Delta V$ , between the surfaces of the capacitor. The unit of capacitance (coulombs per volt) is called the farad.

$$C = \frac{Q}{\Delta V} \quad (1)$$

### 5.3 Effective Capacitance

In this part of the lab, the effective capacitance ( $C_{eff}$ ) for capacitors connected in series and parallel is measured and the results compared to the theoretical values obtained from the formulae:

Parallel Connection:

$$C_{eff} = C_1 + C_2 \quad (2)$$

Series Connection:

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \quad (3)$$

where  $C_1$  and  $C_2$  are the values of two capacitors.

### **Procedure**

Familiarize yourself with the LCR meter (an instrument to measure inductance ( $L$ ), capacitance ( $C$ ) and resistance ( $R$ )).

Select two different fixed value capacitors that differ in capacitance by less than a factor of ten and measure their individual capacitance. Connect the two capacitors in series on the miniature breadboard and measure the effective capacitance. Then connect the capacitors in the parallel configuration and measure the effective capacitance.

### **Data**

Record the capacitance of the individual capacitors and the effective capacitance when they are connected in the series and parallel configurations.

### **Analysis**

Compare the measured effective capacitance for both the series and parallel connections to the theoretical values.

## 5.4 Geometry and Capacitance

The capacitance is actually independent of the charge on the plates and the voltage between them. It is only a function of the geometry of the capacitor. In this part of the lab, the capacitance will be determined from the geometry of the capacitor and then compared to its actual value measured with the LCR meter.

In order to get an accurate calculated capacitance, we have to accurately determine the geometry of the capacitor by measuring certain dimensions. A tool commonly used to make accurate measurements of length is the caliper, or vernier. Figure 1 shows how to use the vernier scale.

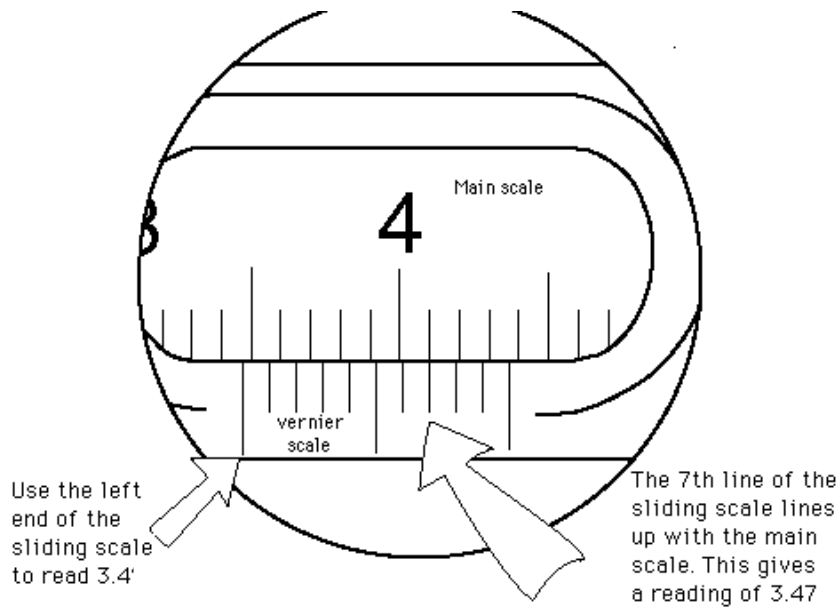


Figure 1: Schematic showing how to read a vernier caliper measurement. Note that since the left end of sliding part lies between 3.4 and 3.5, we read off 3.4. The third digit is obtained from noting that the 7th line of vernier scale aligns almost perfectly with the main scale. Therefore the reading is 3.47cm.

### 5.4.1 Parallel Plate Capacitor

In this case, the capacitance is given by

$$C = \kappa \epsilon_o \frac{A}{d} \quad (4)$$

where  $d$  and  $A$  are the plate separation and area, respectively;  $\kappa$  is the value of the dielectric constant between the plates;  $\epsilon_o$  is the electric permittivity.

## Procedure

Acquire a movable plate capacitor, select an acrylic insert, and measure its thickness using a caliper. Insert the acrylic between the two movable circular plates and close the plates until they contact the acrylic insert and hold it securely. Using the LCR meter, measure the capacitance with the insert between the two circular plates. Carefully remove the inserted acrylic, being sure not to change the distance between the plates and measure the capacitance when the dielectric between the plates is only air. Repeat for a total of 7 plate thickness separations. Assume that the properties of the acrylic are the same for all pieces. Measure the diameter of one of the circular movable plates –you may assume they are identical in size.

## Data

Record the thickness of each of the acrylic pieces used. Make a table showing the value of the capacitance and the distances between the plates with and without the dielectric.

## Analysis

Determine the dielectric constant of the acrylic used,  $\kappa$ . For this purpose, use the measurements of the acrylic's capacitance,  $C_{acrylic}$ , and the capacitance of air,  $C_{air}$  that were obtained by varying the distance between the plates.

Start by Plotting the data in your table:  $C_{acrylic}$  versus  $C_{air}$ . Use this plot and Equation 4 to determine the dielectric constant,  $\kappa$ , for acrylic. Explain your work.

### 5.4.2 Cylindrical Capacitor

It can be shown that in this case the capacitance is given by

$$C = \frac{2\pi\kappa\epsilon_o L}{\ln\left[\frac{b}{a}\right]}, \quad (5)$$

where  $L$  is the length of the cylinder,  $a$  is the inner radius of the capacitor,  $b$  is the outer radius of the capacitor, and  $\kappa$  is the dielectric constant for the material between the cylinders –see Figure 2. Before measuring  $a$  and  $b$ , try answering question 1 in section 5.5.

### Procedure

Use a caliper to measure the inner and outer diameters of the cylindrical capacitor. Measure the length of the capacitor with a meter stick. Measure the capacitance using the LCR meter.

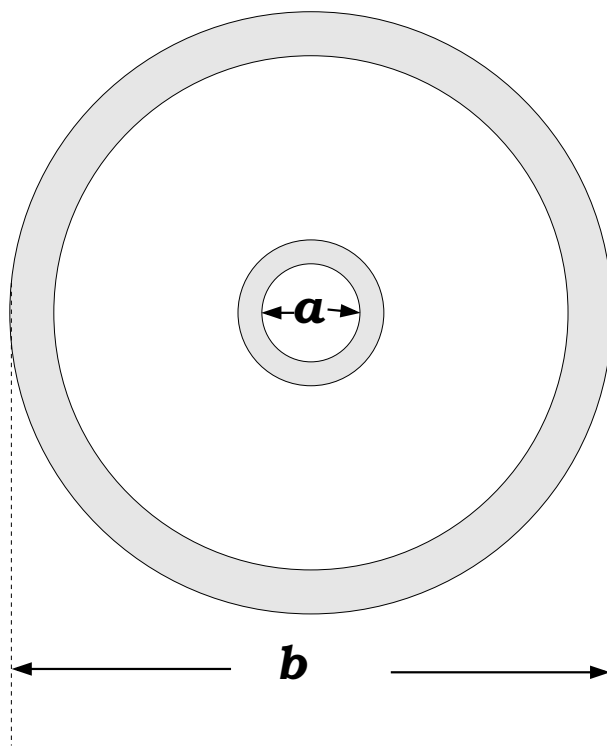


Figure 2: Cross section of the Cylindrical capacitor.

### Data

Record the capacitance, the values of the inner and outer diameters, and the length of the capacitor.

### Analysis

Based on the measured geometric dimensions of this capacitor, determine the predicted value for its capacitance. Compare this value to the measured value.

### 5.5 Questions

1. Use Gauss's Law to outline the derivation of the concentric cylinder capacitor without dielectric  $-eq(5)$ .

2. Suppose that you suspect that the dielectric constant for air is not really one; describe how would you use your data from section 5.4.1 to determine  $\kappa_{air}$ . Assume that the movable metallic plates have a perfect circular shape.

## 6 Ohm's Law

In this lab, the relationship between the voltage across, and the current flowing through, a so-called Ohmic material is explored. The measuring techniques developed in this lab will be essential in the following labs.

### 6.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

### 6.2 Introduction

In this lab, Ohm's law is explored. This law is a relationship between the electric potential difference across a resistor and the current flowing through it. Since resistors are ubiquitous elements of electrical circuits, a solid first hand knowledge of Ohm's law is fundamental.

Potential difference is often referred as voltage and represented as  $V$ . However, to emphasize that voltage is a difference in potential between two points, the symbol  $\Delta V$  will be used to denote it. In the case of a resistor, the voltage across it refers to the difference in potential between its terminals.

Note that in some contexts, it is convenient to have a reference value for the potential and measure the potential difference with respect to that reference. Usually, the reference value is arbitrarily set to zero and called ground. In this context when people speak of voltage at a given point, it is usually understood that such voltage refers to the difference in potential between the given point and ground.

For some materials, the voltage across them is proportional to the current  $I$ , flowing through them. That is

$$\Delta V = IR, \tag{1}$$

where the constant of proportionality,  $R$ , is the resistance of the material. It is a constant in the sense that it does not depend on voltage or current. However,  $R$  may depend on the type of material, shape, temperature, etc. Equation (1) is known as Ohm's law and materials that obey this law are called ohmic. This lab is limited to the study of ohmic materials, like metals and carbon resistors. Resistance is measured in ohms, which are denoted by the Greek letter  $\Omega$ , capital omega. It can be seen from Equation (1) that  $1\Omega = 1VA^{-1}$ . The resistance of metals is generally very low and therefore they are good conductors. In contrast, carbon resistors have a large range of values typically from milli to mega ohms. Often, the

analysis of circuits that use carbon resistors joined by metallic contacts neglects the metallic resistance of the connections.

Ohm's law will be investigated using a DC (direct current) power supply and a DMM (Digital Multimeter) for measuring resistance, current and potential difference. Before beginning the lab, familiarize yourself with the basic usage of these instruments. Regarding the DC power supply, keep in mind that it has polarity. Hence, the current flows from the positive terminal into the load and then comes back to the power supply through the negative terminal. This means that cables connected backwards will give a potential difference with the opposite sign.

Ask your instructor if you have questions about the operation of these instruments.

This lab is divided into three parts. In the first, the applicability of Ohm's law to common carbon resistors is examined. In the other two, Ohm's law is applied to examine two common configurations of carbon resistors, namely parallel and series.

### **6.3 Common Resistor**

In this part, the relationship between voltage and current across a common carbon resistor is studied. The applicability of Ohm's law to these resistors is examined next. A DC power supply is used to vary the potential difference across the resistor and observe how much current flows through it in response.

#### **Procedure**

Obtain three resistors of nominal values:  $1\text{ k}\Omega$ ,  $5\text{ k}\Omega$ , and  $10\text{ k}\Omega$ . Using the DMM, measure the actual resistances and record them. Install a resistor of about  $1\text{ k}\Omega$  into the screw-down board. Use the DMM to set the power supply to about  $1\text{ V}$  and connect it to the resistor. Measure the actual potential difference across the resistor and record it. Turn off the power supply, but do not change its settings. Set the DMM to measure current and connect it in series with the resistor. Turn on the power supply, measure the current flowing through the resistor (and the DMM), and record it. Repeat your measurements for power supply voltages between  $1\text{ V}$  and  $12\text{ V}$ , with increments of about  $2\text{ V}$ . Collect the same kind of data for the other two resistors.

#### **Data**

Organize the data for each resistor in a table that include headings and units.

## Analysis

Determine if your data supports Ohm's law for your resistors. For each resistor, make a voltage vs. current plot, use a ruler to draw the best fit line for your data, and then determine its slope. Write the equations for these lines and compare the slope to the resistance value measured for the corresponding resistor.

Combine all of your plots, including the fitted lines, in the same graph using the same scale. Mark the data for each resistor with a different symbol and add an appropriate legend. Include this graph in your report as part of the analysis.

## 6.4 Resistors in Series

Most resistors in a typical circuit are carbon resistors that follow Ohm's law. In this part, Ohm's law is used to study resistors connected in series. The distinctive feature of this connection is that, in steady state, conservation of charge demands that the same current flows through each resistor. On the other hand, the voltage across each resistor may be different, but it is related to the current by Ohm's law.

### Procedure

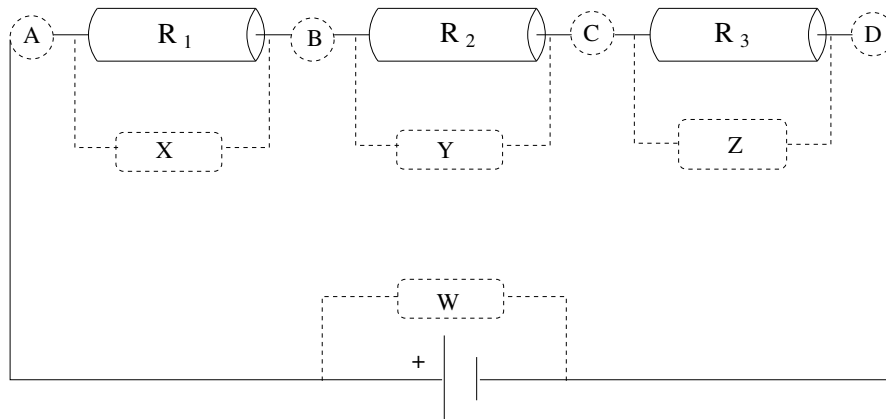


Figure 1: Resistors in series. The circles marked A, B, C, and D show the locations for current measurements. Rectangles marked W, X, Y, and Z show the locations for voltage measurements.

Refer to Figure 1 for circuit and meter setup.

Use resistors with nominal values of  $R_1 = 1 \text{ k}\Omega$ ,  $R_2 = 5 \text{ k}\Omega$ , and  $R_3 = 10 \text{ k}\Omega$  (or any combination given by the instructor). Measure the resistance of each resistor. Using short banana cables, connect the resistors

in series. Using your multimeter, set the power supply to 10 V (open circuit voltage). Connect the power supply with the array of resistors and measure the voltage at the terminals of power supply (loaded voltage).

Proceed to measure the voltage across each resistor (DMM in parallel with resistor at locations X, Y, Z).

Set the DMM to measure DC current in mA (milli-amperes) and insert it in series at location 'A' to measure the current flowing from the power supply into  $R_1$ . In a similar fashion measure the current flowing from  $R_1$  into  $R_2$  (location B), the current flowing from  $R_2$  into  $R_3$  (location C), and the current flowing from  $R_3$  back into the power supply (location D).

### Data

For each resistor, measure and record its resistance, voltage across and current flowing through it. Also, the current flowing into the circuit and the voltage across the terminals of the power supply (both open circuit and loaded values). Organize the measurements associated with the individual resistors (resistance, current and voltage) in a table with headings that include units.

### Analysis

Use Ohm's law to calculate the resistance of each resistor from your voltage and current measurements. Using the same technique, calculate the effective resistance of the circuit. Compare all calculations with your resistance measurements. Add this information to your data table and turn it in as part of your analysis.

## 6.5 Resistors in Parallel

In this part another common configuration, the parallel connection, will be studied using the same resistors as in Section 6.4. The distinctive characteristic of this connection is that, in steady state, conservation of energy demands the same voltage across each resistor. On the other hand, the current flowing through each resistor may be different, but it is related to the voltage by Ohm's law.

## Procedure

Using short banana cables, connect the resistors in parallel, using Figure 2 as a guide. Set the power supply to 5 V (open circuit) and then connect it to the circuit. Measure the loaded value of the voltage across the terminals of the power supply (location W). Also, measure the voltage across each individual resistor (locations X, Y, and Z).

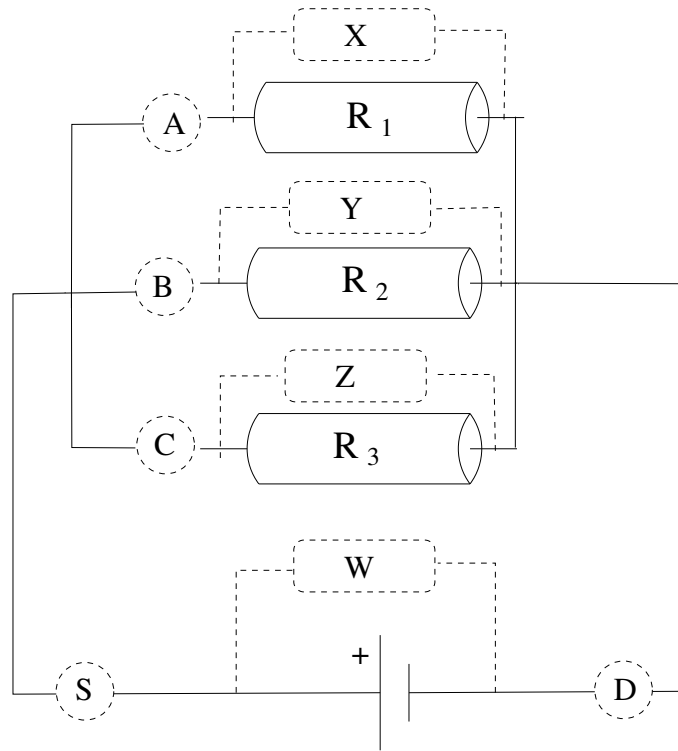


Figure 2: Resistors in parallel. The circles marked A, B, C, and D show the locations for current measurements. Rectangles marked W, X, Y, and Z show the locations for voltage measurements.

Set the multimeter to measure DC current in mA (milli-ampere). Measure the current through each resistor (locations A, B, and C) as well as the current going back into the power supply (location D).

## Data

Organize your data in a table similar to the one created for the connection in series, Section 6.4.

## Analysis

Use Ohm's law to calculate the resistance of each resistor and the effective resistance of the circuit from your voltage and current measurements.

Compare with your resistance measurements. Add this information to your data table and turn it in as part of your analysis.

## 6.6 Questions

1. Given your measured values for  $R_1$ ,  $R_2$ , and  $R_3$ , what does the theory predicts for their equivalent resistance when they are connected in series?
2. Given your measured values for  $R_1$ ,  $R_2$ , and  $R_3$ , what does the theory predicts for their equivalent resistance when they are connected in parallel?

## 7 Kirchhoff's Laws

This lab explores the Kirchhoff's loop and node laws using direct current circuits. In this context, we also examine the voltage divider, a simple but widely used circuit.

### 7.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

### 7.2 Introduction

Kirchhoff's loop and node laws are fundamental for the study of circuits. The loop law states that the algebraic sum of all potential differences around a closed loop is zero, which follows from conservation of energy. The node law states that the current flowing into a circuit node is the same as the current flowing out of it and this follows from conservation of charge.

Kirchhoff's laws are explored using direct current, DC, circuits. The lab is divided in three parts. In the first two, Kirchhoff's laws are tested. In the last, the laws are used to examine a simple but widely used circuit: the voltage divider. This lab assumes that the student already knows how to measure current and voltage using a DMM and has some experience using DC power supplies. Ask your instructor if you need help.

### 7.3 Kirchhoff's Loop Law

Conservation of energy applied to a closed branch of a circuit (a loop) implies that the algebraic sum of potential differences across each element in the loop must be zero. This statement is one of the basic concepts in the study of electrical circuits and will be tested in this part of the lab.

#### Procedure

Build the circuit as shown in Figure 1. Set voltage of the power supply to 5 V. Measure and record the potential difference across each resistor in the circuit. Recall that the DMM must be connected in parallel (across the element).

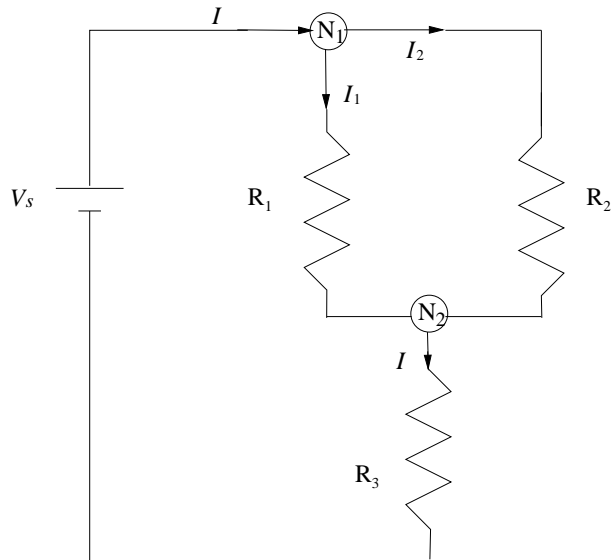


Figure 1: Test circuit: This figure shows the setup for Sections 7.3 and 7.4. Use the following values (or those provided by your instructor):  $R_1 = 10\text{k}\Omega$ ,  $R_2 = 5\text{k}\Omega$ , and  $R_3 = 1\text{k}\Omega$ .

### Data

Your data should be organized in a table listing the measured value of resistances and potential difference with appropriate units.

### Analysis

Examine whether or not your data support the Kirchhoff's loop law. Start by writing the algebraic expression for Kirchhoff's loop law for the loop that contains the power supply,  $R_1$ , and  $R_3$ . Proceed similarly for the other two possible loops in Figure 1.

## 7.4 Kirchhoff's Node Law

Conservation of charge applied to any node in a circuit implies that the algebraic sum of all currents at the node must be zero. This statement is another key concept in the study of electrical circuits and will be tested next.

## Procedure

Using the circuit shown in Figure 1, measure the current flowing in and out of node  $N_1$  and record your measurements. Recalling that the DMM must be connected in series (with a circuit element), you must break the circuit at the points of measurement to insert the multimeter. For  $N_1$  you would want the meter to replace the wire between  $V_s$  and  $N_1$  for one measurement,  $N_1$  and  $R_1$  for another, and  $N_1$  and  $R_2$  for the final measurement. Before starting this, mark on the schematic in this manual where you think the meter should be inserted and check it with your lab TA. Don't forget to make sure the DMM is set and wired to measure DC current.

Repeat the above procedure for  $N_2$ . Where do you need to measure the current for measurements associated with  $N_2$ ? Do you already have any of these measurements?

## Data

Make a table of the currents you measured. Be clear about the location of each measurement.

## Analysis

Examine whether or not your data support the Kirchhoff's node law. Start by writing the algebraic expression for Kirchhoff's node law for node  $N_1$  in Figure 1.

## 7.5 Voltage Divider

Voltage dividers are one of the most widespread electronic circuit fragments, typically used when one portion of a circuit requires a smaller voltage than the input voltage. The goal of the voltage divider, shown in Figure 2, is to provide an specific voltage between points A and B, which is obtained by carefully choosing  $R_a$  and  $R_b$ . The potential difference across each resistor can be obtained by applying the loop law for voltages around the circuit, and the node law to currents at the junction of the resistors (point A) along with Ohm's law to the individual resistors. The result is

$$V_{MA} = \frac{R_a}{R_a + R_b} V_s \quad (1)$$

and

$$V_{AB} = \frac{R_b}{R_a + R_b} V_s \quad (2)$$

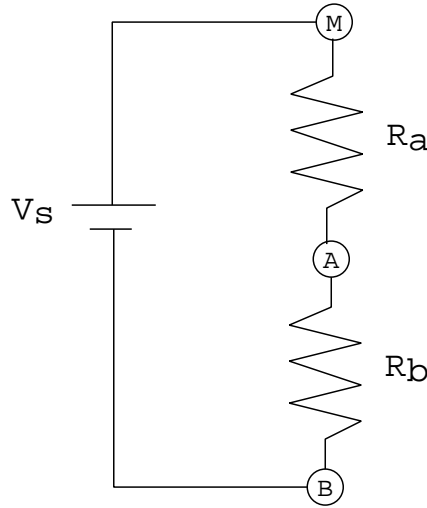


Figure 2: Voltage Divider.

The value of  $R_b$  can be obtained from Equation 2, given the input voltage,  $V_s$ , and the required  $V_{AB}$  as

$$R_b = \frac{V_{AB}}{V_s - V_{AB}} R_a. \quad (3)$$

This equation is the starting point in the selection of a resistor pair. For example if you need to reduce the input voltage from 0 – 100 volts to 0 – 1 volts, Equation 3 gives  $R_b = R_a/99$ , which (for many practical purposes) is  $R_b = R_a/100$ . (When considering resistor tolerances and availability.) It should be clear that the required reduction could have been obtained using various resistor pairs (e.g.  $R_a \approx 100 \Omega$  and  $R_b \approx 1 \Omega$ ).

**Power Considerations** Another important consideration in choosing the pair is power dissipation. For instance, choosing  $R_a \approx 100 \Omega$  and  $R_b \approx 1 \Omega$  for an input voltage of 100 V results in a power dissipated by  $R_a$  of

$$P_{R1} = \frac{V_{MA}^2}{R_a} = \frac{(99 \text{ V})^2}{100 \Omega} \approx 100 \text{ W} \quad (\text{watts}) \quad (4)$$

Fortunately, the power supply will not put out 100 Volts, so there is no need to worry about ‘fried’ resistors and possibly burned fingers.

The resistors you are using are probably rated at 0.25 W and this rating is greatly exceeded when the input is 100 V. To solve this problem, you could use resistors rated at 100 W or use a different resistor pair. Note that the largest voltage drop is across the large resistor,  $R_a$ , and the power dissipated in the circuit is almost all dissipated by it.

### **Procedure**

Select resistors with nominal values of  $R_a = 10\text{ k}\Omega$  and  $R_b = 100\ \Omega$ . Measure their actual resistances and record them. Construct the circuit shown in Figure 2 with the voltage supply set to 10 V. Measure the potential differences across each resistor and record your values.

### **Data**

Organize your data in a table with headings that include units.

### **Analysis**

As part of your analysis, derive Equations 1 and 2. Make a sketch of Figure 2 showing all your measurements and analysis results. Include this sketch in the analysis section of your report. Also include the power dissipated in each resistor.



## 8 Magnetic Forces

This experiment investigates the magnetic field produced by an electrical current and the force exerted on moving charge by a magnetic field. These effects are described respectively by Ampere's Law and the magnetic force law. Furthermore, a hands-on demonstration of applying the right-hand rule is provided.

### 8.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

### 8.2 Introduction

The experiment is divided into two parts. In the first part, a qualitative feel for the magnetic field produced by a current along a straight wire and for the force on such current by a permanent laboratory magnet is developed. The second part involves a quantitative determination of the magnetic force between parallel wires using a current balance.

The magnitude of the magnetic field produced by a current  $I$  passing through a long straight wire is given by

$$B = \frac{\mu_0 I}{2\pi r}, \quad (1)$$

where  $r$  is the radial distance from the wire and  $\mu_0$  is the permeability of free space. In SI units,  $\mu_0 \equiv 4\pi \times 10^{-7}$ . The units of  $\mu_0$  are left as an exercise.

The direction of the magnetic field is perpendicular both to the direction of  $I$  and to the radius vector  $\vec{r}$ . The result of this is that the magnetic field lines circle the wire according to the right-hand rule. (With the thumb of your right hand pointing in the direction of  $I$ , your fingers curl in the direction of  $\vec{B}$ .) The strength of the field decreases with distance from the wire as  $1/r$ .

When two current-carrying wires are placed parallel to each other, the magnetic field from one wire produces a force on the current in the other wire, and vice-versa. The law describing the force that a magnetic field exerts on moving charge, in particular on a current  $I$  flowing through a wire of length  $\ell$ , is given by

$$\vec{F} = I(\vec{\ell} \times \vec{B}). \quad (2)$$

When the two wires are parallel, and  $\vec{B}$  and  $\vec{\ell}$  are mutually perpendicular, the cross product is just the product of the magnitudes,  $F = I\ell B$ . Substituting for  $B$  from the other wire, one obtains that

$$F = \mu_0 \frac{I^2 \ell}{2\pi d} = \left( \frac{\mu_0 \ell}{2\pi d} \right) I^2, \quad (3)$$

where  $d$  is the spacing between the wires. This expression is valid as long as  $d \ll \ell$ . The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

### 8.3 Qualitative Observations

**Magnetic field lines.** Use a compass to investigate the magnetic field produced by a current in a straight wire. A compass needle orients itself with magnetic field lines, as indicated in Figure 1. The north pole of the compass points in the same direction as the field. **Note:** on our compasses, magnetic north is indicated by the red end of the needle.

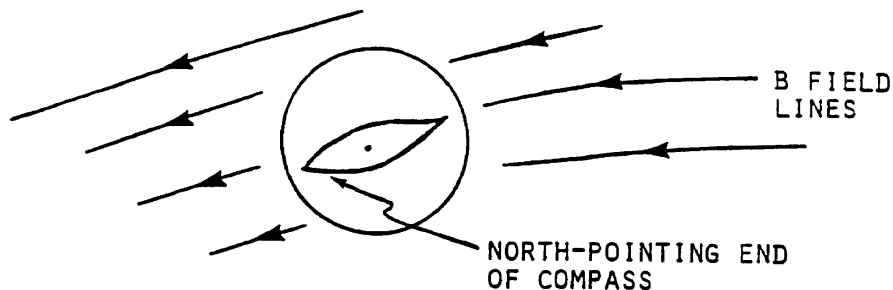


Figure 1: Diagram of the system for testing Ampere's Law.

The magnetic field is generated by passing a large current through a vertically mounted wire segment. The value of  $I$  is determined by measuring the voltage which it produces in passing through a piece of metal, called a 'shunt', that has a very low resistance. The resistance is such that the voltage is 1 mV per ampere of current.

Turn on the power supply and measure the voltage across the shunt resistor. Record its value and use it to calculate the current. Include your findings in your report.

Explore the  $B$  field surrounding the wire and sketch out the pattern of the magnetic field lines. Also, use the compass to determine if the field goes around the wire in the direction predicted by the right-hand rule. Give a short description of the results in your write-up.

**Earth's magnetic field.** By experimenting with the compass, find the point along the meter stick at which  $B$  due to both wires is equal to the earth's magnetic field,  $B_{\text{earth}}$ . Can you figure out a way of doing this? Measure the distances  $r_1$  and  $r_2$  from the wires to the point where the fields are equal. Use Equation 1 to calculate the magnetic field due to each wire. Being careful to account for the direction of these fields, superimpose the wire's fields (add them) to obtain  $B_{\text{earth}}$ . The SI unit for magnetic field is the tesla (T). Compare your result with the known value for  $B_{\text{earth}}$  of about  $3 \times 10^{-5}$  T.

**Permanent magnet.** Examine the force which a permanent magnet produces on the current. Remove the 'keeper' from the magnet (keep any compasses well away) and hold one of the wires from the power supply between the poles of the magnet. Turn on the power supply and feel the force on the wire. From the direction of the force, the direction of the current, and remembering that field lines come out of the north pole and go into the south pole of a magnet (as in Figure 2) determine which pole of the magnet (labeled A and B) is the north pole. Make a sketch of the situation and include it in your report.

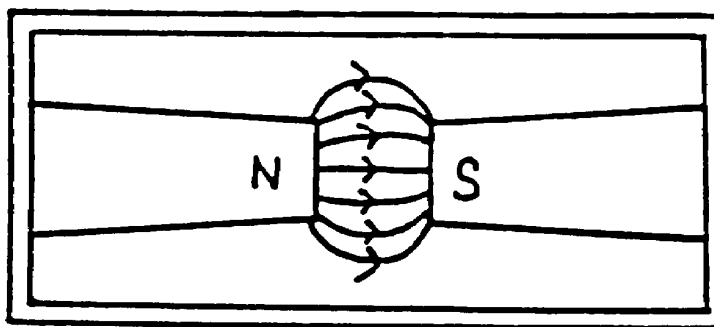


Figure 2: The magnetic field lines between the poles of a strong magnet.

## 8.4 Current Balance

In this part, we make a quantitative measurement of the force between two parallel wires carrying equal currents. The force is measured using a specially designed apparatus called a current balance. This apparatus balances the magnetic force against the gravitational force provided by small weights. The measurements are used in two ways:

1. To test if the force is proportional to  $I^2$ , as predicted by Equation 3.
2. Estimate the value of  $\mu_0$  from the constant of proportionality

$$C' = \mu_0 \ell / 2\pi dg \quad (4)$$

This will be approximate because it is difficult to accurately measure the wire separation  $d$ .

The experimental setup is shown in Figure 3. A laser beam is reflected onto the wall by a small mirror attached to the rotating balance. The long 'lever arm' of this arrangement enables even small departures from balance to be detected as a deflection of the laser beam on the wall. The deflection depends on the amount of mass used. We counteract the gravitational force by increasing the magnetic force between the wires. Hence, to bring the laser beam back to its initial position, we increase the current. This procedure allows us to determine how  $F$  varies with  $I$  for constant wire separation  $d$ .

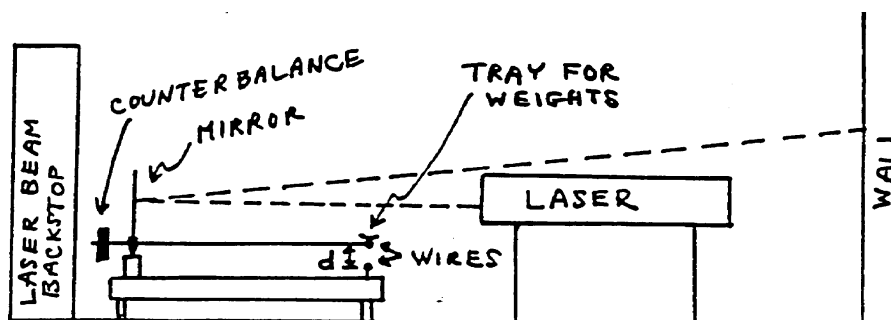


Figure 3: Current balance setup.

The circuit for providing current to the balance is shown in Figure 4. As long as the current in the two wires is oppositely directed, it does not matter what its sign is. Therefore, a source of alternating current or an AC voltage can be used. The variac is a variable transformer that enables the alternating voltage to be manually adjusted, and the flexiformer reduces the voltage to safe levels. The current value is read by an AC ammeter.

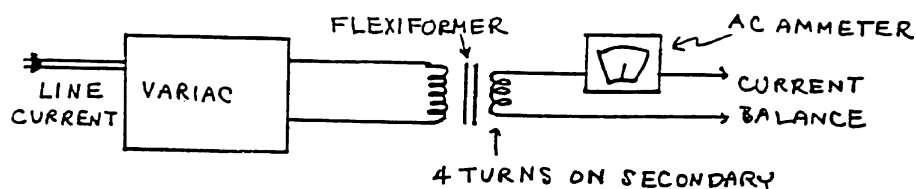


Figure 4: Circuit for supplying current to the balance.

### Procedure

**Safety Note:** LASERS used in this laboratory are all class II. Treat them with respect! They will not hurt your skin, but a sustained direct or reflected beam can cause permanent eye damage!

To begin, check that you understand the wiring of the circuit, for example, that the current does indeed flow in opposite directions in the two wires.

A diagram can be helpful here. Make sure that the balance moves freely without binding. The two wires should be about 0.5 cm apart (center-to-center) with no current flowing and no weights in the pan. The movable wire should be directly above the fixed wire. With no current applied to the balance, mark the position where the laser beam hits the paper on the wall.

Next, turn the variac on and carefully increase the current to see if the balance responds. Check that the laser beam returns to its initial position when the current is turned off. Once you have gone through these checks, be careful not to disturb the balance, or you have to start your measurements over.

Obtain a set of observational data by placing one of the small weights in the pan. Adjust the current with the variac until the laser beam returns to its initial spot, then read the current. Repeat several times with additional weights. Increments of about 50 mg work well (covering the available current range).

When you are done, check that the balance returns to its initial position with all the weights removed and the current turned off. As accurately as you can, measure  $d$ , the center-to-center distance between the two wires, and  $L$ , the length through which current flows.

### Data

Record the value of the center-to-center distance between the wires,  $d$ , their length  $L$ , and the estimates for their uncertainties. Report in a table the masses used and the corresponding measurements of the current required to balance each mass.

### Analysis

Make a plot of the mass  $m$  versus  $I^2$  to see if the data make sense. Since the gravitational force is proportional to  $m$ , your points should lie along a straight line that passes through the origin. The slope should be proportional to the constant  $C'$ . Use a ruler to draw the best fit line to your data and determine the slope and the value of  $C'$ . From this, calculate  $\mu_0$  and compare it to its actual value.

## 8.5 Questions

1. What is the shunt current?
2. What are the units of  $\mu_0$ ?
3. Make a Free Body Diagram for the mass supported by the balance.

## 9 Magnetic Forces II

In this lab, we investigate the magnetic force on individual charges. In the process, we estimate the charge-to-mass ratio for electrons.

### 9.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

### 9.2 Introduction

To estimate the charge-to-mass ratio for electrons,  $q_e/m_e$ , we use the magnetic field produced by two identical coils known as Helmholtz coils, see Figure 1. In this configuration, the current circulates in the same direction in both coils, and the resultant magnetic field is fairly uniform close to the axis of the coils, around the mid-point between them.

The magnitude of the magnetic field along the axis, exactly at the mid-point, can be easily calculated by applying Biot-Savart law to a single loop carrying a current,  $I$ , and then superimposing the results for the  $N$  loops on each coil. The result is

$$B = \frac{16\mu_0}{\sqrt{125}} \left( \frac{NI}{D} \right), \quad (1)$$

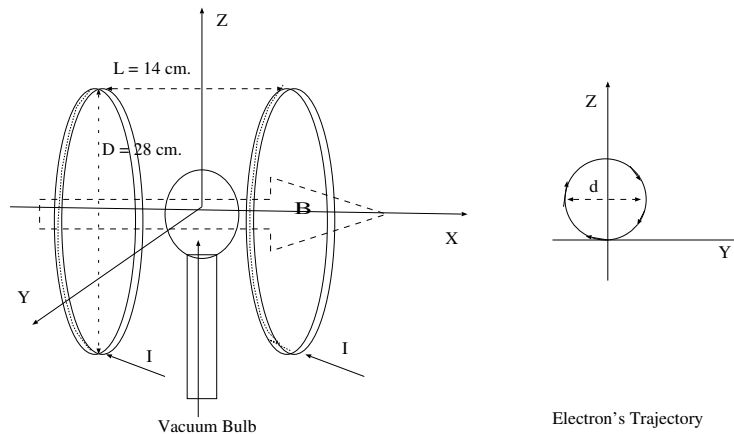


Figure 1: Instrument Setup. The magnetic field is generated by two identical coils; each coil has 140 turns. In this configuration, the magnetic field is fairly uniform at the mid-point between the coils. The vacuum tube contains a rarefied inert gas and has an electron gun that controls the velocity of the electrons. The inset on the right depicts the electron's track on a plane perpendicular to the coil's axis at the mid-point between them—the magnetic field is entering the page.

where  $\mu_0 \equiv 4\pi \times 10^{-7} \text{TmA}^{-1}$  is the permeability of free space,  $I$  is the current on each loop of the coils,  $N$  is the number of loops per coil, and  $D$  is the coil's diameter. The field's direction is parallel to the axis of the coils; we assume the field to be uniform in the vicinity of the mid-point between the coils.

In a previous lab, we have seen that a current-carrying wire feels a force due to a magnetic field – presumably due to the interaction between charges moving in the wire and the magnetic field. In this lab, we examine the motion of a stream of free electrons under the influence of a magnetic field.

The magnetic force on individual charges is given by

$$\vec{F} = q\vec{v} \times \vec{B}, \quad (2)$$

where,  $\vec{B}$  is the magnetic field acting on a charge  $q$  moving with velocity  $\vec{v}$ .

Even when there are no wires to confine the electrons, we are not going to be able to see the actual electrons. However, the trace that they leave behind as they collide with gas molecules is visible. To visualize the electron's trajectories, we place a vacuum tube at the mid-point between the Helmholtz coils, see Figure 1. This tube contains a low-pressure inert gas and an electron gun that can shoot electrons with a given kinetic energy and direction. The tube is aligned so the electron's velocity is perpendicular to the magnetic field produced by the Helmholtz coils.

Inside the electron gun, electrons that have broken free from a hot filament are accelerated by a potential difference,  $\Delta V$ . Assuming that the electrons start from zero velocity, the change in their kinetic energy is equal to the work done on them by the potential difference,  $\Delta V$ . That is

$$q_e \Delta V = \frac{1}{2} m_e v_e^2, \quad (3)$$

where  $q_e$ ,  $m_e$ ,  $v_e$  are the charge, mass, and speed of the electron.

Once an electron leaves the electron gun, it finds itself immersed in a uniform magnetic field, whose magnitude is given by Equation (1). It turns out that the electrons trace circles, whose radius depends on the magnitude of their velocity and strength of the magnetic field. The motion of electrons moving at speeds much smaller than the speed of light (non-relativistic electrons) is described by Newton's second law,  $\vec{F} = m_e \vec{a}$ . For circular motion, the acceleration is given by  $\vec{a} = (v^2/r)\hat{r}$ , where  $v$  is the speed of the particle,  $r$  the radius of the circle and  $\hat{r}$  is a unit vector pointing radially inwards. In our setup, the electron velocity is perpendicular to the magnetic field. Therefore, the equation of motion for the electrons can be written as

$$q_e v_e B = m_e \frac{v_e^2}{r}. \quad (4)$$

Furthermore, the B field and the electron's velocity are given by Equations (1) and (3), respectively. Substituting those equations into (4), we obtain an expression for the electron's charge-to-mass ratio.

$$q_e/m_e = \frac{2\Delta V}{B^2 r^2}, \quad (5)$$

where  $\Delta V$  is the electron's accelerating voltage,  $r$  is the radius of circle traced by the electrons, and  $B$  is given by Equation (1).

### Procedure

#### WARNING

*The instrument that we are going to use is shown in Figure 1.*

*Do not use this instrument continuously for more than one hr.*

*Pay special attention to the startup and shutoff sequences.*

*Also after each measurement, return the accelerating voltage to zero.*

Before turning on the main switch (Red Switch), make sure that all switches are in their off position, and all the control knobs are in their start positions (lowest value).

Once the main switch is on, an orange glow from the filament at the electron gun should be visible; before continuing, wait about 5 minutes for the filament to get to the operating temperature.

Next, increase the accelerating voltage to about 100 V. A blue ray exiting the electron gun should now be visible – this is the track left by the electrons as they collide with the atoms inside the tube. Turn on the current on the Helmholtz coils and increase it to about 1.5 A. Observe the new trajectory of the electrons. The new track should be a circle. Since the Earth's magnetic field may interfere with the electron's track, use a magnetic compass to align the instrument's magnetic field with the horizontal component of the Earth's magnetic field. If after doing this alignment, the electron's track looks more like a helix than a circle, ask your TA to align the vacuum tube –to avoid leaving any residual grease on the tube, gloves are required for this operation. After the alignment, reset the current and accelerating voltage to zero.

Once your apparatus is set up, you can start collecting data. Set the current at 1.5 A, and vary the accelerating voltage from 100 V to 250 V in increments of 50 V. For each of these voltages, measure and record the diameter of the electron's track. (After each measurement reset the accelerating voltage to zero.)

Increase the current to 2 A, and vary the voltages as before, recording the track diameter for each voltage.

After finishing collecting data, reset the current and accelerating voltage to zero. Set the switches of the accelerating voltage and current to their off position; finally set the main switch to its off position.

**Note:** *should you need to repeat your measurements, then you must repeat the start and alignment procedures for the instrument.*

### Data

Organize all your data (current, voltage, and trajectory diameter) in a table with descriptive headers that include units. The first column should be an index to indicate the sequence of your measurements.

Record the coil's mean diameter and the number of loops,  $N$ , per coil.

### Analysis

For each current value, calculate the magnetic field near the electron's track.

Using the accepted value for the mass and the charge of the electron, calculate the speed of the electrons for each accelerating voltage. Write your response as a fraction of the speed of light. Incorporate your results into your data table.

Calculate the value of electron's charge-to-mass ratio,  $q_e/m_e$ , for each measurement. Incorporate your values into your data table.

Plot the sequence of  $q_e/m_e$  values against their index in your data table.

From your data, calculate the average value of  $q_e/m_e$ .

Trace a horizontal line on your plot to represent the average value of  $q_e/m_e$ .

Based on your plot, how much confidence do you have on your results?

Calculate the percentage difference between your average and the accepted value of  $q_e/m_e$ .

## 10 Oscilloscopes

This lab is a tutorial on oscilloscopes. It is introduced here because the rest of the labs require you to measure voltages that change in time.

### 10.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand. Also, watch the following video; it should help make your lab go more smoothly.

<https://drive.google.com/file/d/0Bzyx7ueMSaudWVg4bHhjZVpJU00/view?ts=58ffa10b>

### 10.2 Introduction

In this lab, the oscilloscope is introduced as a tool to measure alternating voltages. The lab will guide you through a series of simple exercises to practice the usage of this important measurement device. As such, this laboratory may not require a full report, but you may be required to pass a practice quiz or fill out a worksheet. Follow the directions of your instructor to report your data and observations.

An oscilloscope is a device for measuring and displaying a voltage waveform, either versus time or another voltage. A cathode ray oscilloscope utilizes a vacuum tube similar to that of an old TV set or computer monitor to display the waveforms. A beam of electrons (the cathode ‘rays’) is emitted by a hot electrode at the back of the tube and accelerated through a potential difference of several kilovolts or more toward positive charge on the inside front face of the cathode ray tube, or CRT. The resulting energetic electrons strike a phosphor coating on the front face of the tube, causing the phosphor to emit the light you see. The electron beam is deflected or ‘swept’ across the face of the CRT linearly with time to provide a time base or axis. At the same time, the beam is deflected upward or downward by the voltage being measured. The result is a display of the signal versus time. The electron beam is deflected by electric or magnetic forces inside the CRT.

Refer to Figures (1) and (2) which show the front of the Oscilloscopes used in this lab.

### 10.3 Setup

In this section, we describe how to set up the oscilloscope to help you become familiar with its operation. Begin by Turning on the oscilloscope.

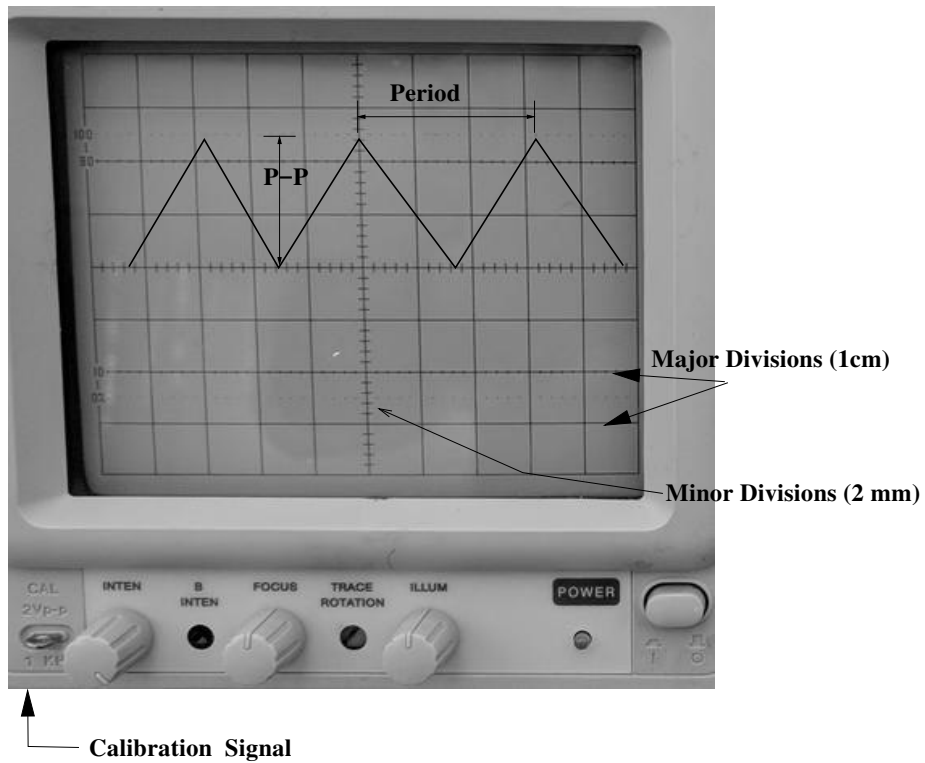


Figure 1: Oscilloscope's Screen.

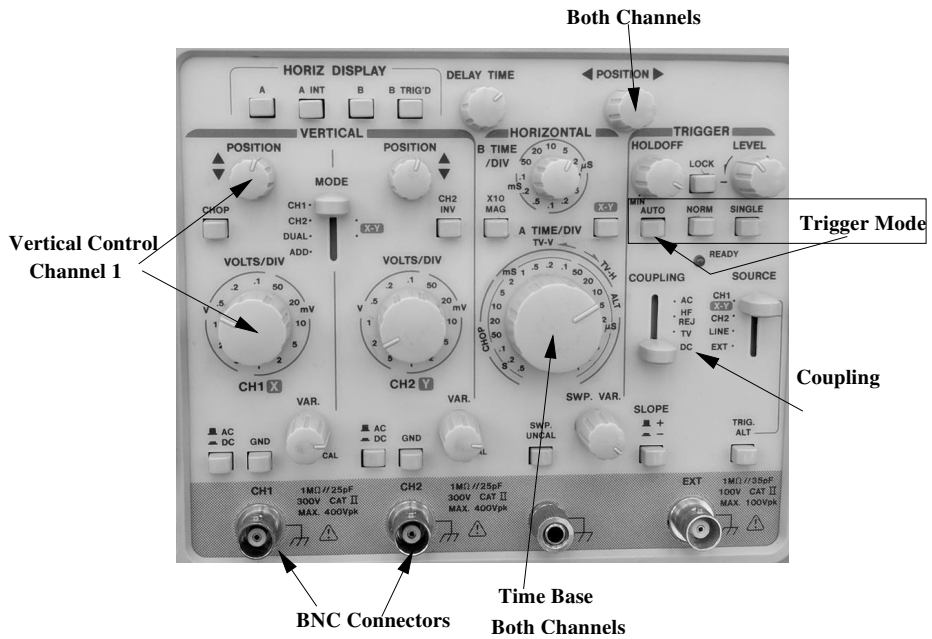


Figure 2: Control Panel of the Oscilloscope.

## Procedure

**Trigger System.** This system is essential to the proper operation of the oscilloscope. At its core, this system tells the oscilloscope when to display a waveform. Basically, it looks at the triggering signal and waits until its amplitude reaches a certain level to trigger the display of the waveform.

For the exercises in this lab, basic setup of this system will suffice. The **Trigger Mode** is set by engaging one of the three buttons AUTO, NORM, or SINGLE. Use the auto mode, so the trigger level is automatically matched to the trigger signal. Also, verify that the LOCK button is engaged, so trigger level is automatically maintained. The COUPLING lever should be set to AC. The SLOPE and ALT buttons should be disengaged. Finally, set the **Trigger Source** to CH1.

**Vertical System.** This system controls the display of the vertical information (Voltage amplitude). It deflects up and down the electron beam according to the input signal.

With this system, we can choose how to connect the signal to each channel of the oscilloscope via the COUPLING lever. We usually use AC coupling, which blocks the DC signal from the input. We can operate this system in various modes: CH1 or CH2, which only display individual channels, DUAL, which display both channels simultaneously, and ADD, which add the signals from both channels. When displaying both channels, you have a choice between normal or chop; when the CHOP button is engaged, the system draws a bit from each channel during the same sweep; if this button is disengaged, then the system draws the signal from one channel during a full sweep and then signal from the other on the next sweep. We rarely use the chop mode, which is appropriate for slow sweep rates.

The vertical system also controls the vertical sensitivity, that is, how many volts per major division on the screen. It also allows you to move the waveform vertically, so you can align the signal with the grid to perform your measurements.

**Horizontal system.** This system controls the deflection of the electron beam horizontally, according to a given sweep rate or time base;

The TIME/DIV knob sets the time per major division on the horizontal grid. This setting controls how fast the sweep rate is for both channels. The POSITION knob allows you to move the waveforms from both channels horizontally and simultaneously, so you can align them with the grid on the screen.

Make sure that the SWP/UNCAL, x10/MAG, and x-z buttons are disengaged;

**Calibration.** To verify the calibration of the oscilloscope, move the VAR

knobs to the cal position; connect the calibration output terminal to Ch 1. Set the vertical MODE lever to CH1 to display only that channel. Adjust the INTEN and FOCUS knobs to make the trace well-focused and easy to see. Adjust the horizontal and vertical POSITION knobs to center the trace on one of the grid lines. Make sure that the calibration signal is correctly displayed on the screen. Repeat for channel two.

**Signal Generator.** Connect the output of the signal generator to the Ch 1 input of the oscilloscope. This connection can be made using the ‘banana’ leads and BNC-to-banana adapters. The outside conductor of the coaxial BNC connector is the ground side and corresponds to the black banana terminal. Make sure that the ground of the signal generator is connected to the ground of the scope. Select a sine wave output from the signal generator. You may need to adjust the trigger LEVEL to obtain a stable display; if necessary also adjust the focus and intensity.

Adjust the signal generator to produce a sine wave of frequency 1 kilohertz (1 kHz). Check if this setting is correct by measuring the period,  $T$ , of the signal with the oscilloscope. For a signal of frequency  $f$ ,  $T = 1/f$ . Adjust the TIME/DIV knob so that each cycle of the sine wave corresponds to one or two-centimeter divisions. Then measure the period of several cycles of the sine wave. Fine tune the frequency so that it is 1kHz, according to the oscilloscope.

Adjust the amplitude of the displayed signal to 2 volts (V) peak-to-peak (p-p), again according to the oscilloscope.

Move the horizontal position to the right so that you can see the beginning of the trace. What are the effects of adjusting the trigger level and the trigger slope?

Sketch the sinusoidal waveform with appropriately labeled axes. Examine and sketch the other signals which the signal generator produces, namely triangle and square waves.

## 10.4 Time Delays

During the semester, you will learn about the physics of resistors, capacitors, and inductors. These components are widely used in electrical circuits. In this lab, we used them to illustrate how the oscilloscope can be used to measure time delays on voltage signals that arise due to the presence of these components.

Consider the circuit shown in Figure 4 with a sinusoidal input signal from the function generator. The output signal observed at the resistor  $R_0$  will have the same frequency as the input signal. However, depending on the type of component used in the circuit, the output signal could have a different amplitude and phase constant,  $\phi_0$ . The oscilloscope allows

us to observe these signals and determine their amplitudes and relative phases. Figure 3 shows an sketch of these kind of signals and their phase differences in the time domain. The time differences can be converted to angles by considering that one full oscillation corresponds to an angle of  $2\pi$  radians or 360 degrees. Hence to convert a time interval like  $\Delta t_A$  to an angle, we could use the following proportions

$$\frac{\Delta t_A}{T} = \frac{\phi_A}{2\pi} = \frac{\theta_A}{360} \quad (1)$$

where  $T$  is the period (in the same units as  $\Delta t_A$ ),  $\phi_A$  is the phase difference in radians, and  $\theta_A$  the same phase difference but in degrees. Also shown in the Figure 3 is the peak-to-peak range of the input signal. It is usually easier to measure peak-to-peak values and divide them by two to get the Amplitude of the signal.

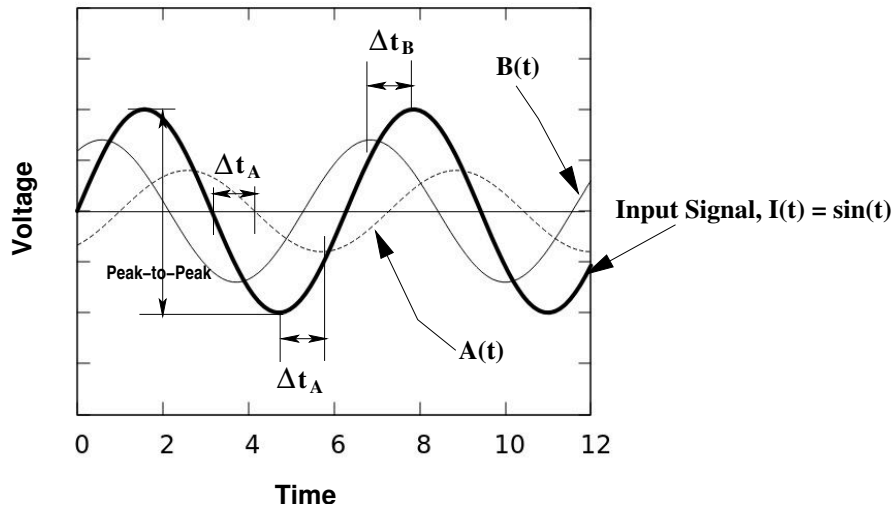


Figure 3: Sinusoidal signals  $I(t) = \sin(t)$ ,  $A(t) = 0.7 \sin(t + 1)$ , and  $B(t) = 0.4 \sin(t - 1)$ ; the phase differences, in the time domain, between  $I(t)$  and the other signals are labeled  $\Delta t_A$ , and  $\Delta t_B$ . The peak-to-peak range for  $I(t)$  is also shown. Note that phase differences may be easier and more accurate to measure between the signals' intersection with a common time axis, rather than between their peaks.

### Procedure

Connect the series Resistor-Resistor circuit of Figure 4 and provide a sinusoidal input to the circuit from your signal generator. Set the frequency of the signal generator to 1 kHz and its amplitude to 12 V (p-p). The input from the signal generator should be connected to Ch 1, while the signal from the RR-circuit is connected to Ch 2 (make sure both channels

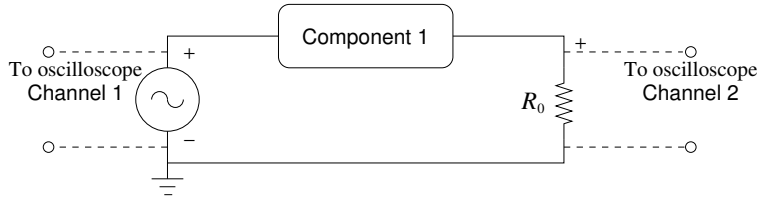


Figure 4: Base Circuit: Component 1 can be a Resistor,  $R$ , a Capacitor  $C$ , or an Inductor,  $L$ . Nominal values for these components are  $R = 1 \text{ k}\Omega$ ,  $C = 100 \text{ nF}$ ,  $L = 65 \text{ mH}$ , respectively. The nominal value for  $R_o$  is  $220 \Omega$ . The Signal generator is set to  $1 \text{ kHz}$  and  $12 \text{ V(p-p)}$ .

are AC coupled). Observe both channels and adjust the vertical Position to bring the peaks from Ch 1 to one of the horizontal grid lines; Adjust the vertical Position of the peaks in Ch 2 to the same level as in Ch 1. Sketch the waveforms with appropriately labeled axes. Can you measure any difference in time between the peaks in Ch 1 and Ch 2? If so, what is it?

Next, turn off the signal generator and replace the resistor  $R$  by a Capacitor of about  $100 \text{ nF}$  (1 nano Farads is equal to  $1 \times 10^{-9} \text{ F}$ ); Keep the same settings on the signal generator and monitor both channels of the Oscilloscope. Bring the peaks from both channels to the same level on the grid.

Is there a time difference between the peaks from Ch 1 and Ch 2 in this case? If so, which channel shows first a peak? Based on this information, is the signal from the RC-circuit ahead or lags behind the input.

Sketch the waveforms from both channels with appropriately labeled axes. Mark the separation between the peaks from Ch 1 and Ch 2; record the value of this time interval in seconds.

Finally, turn off the signal generator and replace the capacitor,  $C$ , with an inductor of about  $65 \text{ mH}$  (millihenries); Keep the same input and monitor both channels of the Oscilloscope. Bring the peaks from both channels to the same level on the grid.

Is there a time difference between the peaks from Ch 1 and Ch 2? If so, does the signal from the LR-circuit lags or is ahead of the input signal?

Sketch the waveforms from both channels with appropriately labeled axes. Mark the separation of the peaks between them; record the value of this time interval in seconds.

## 11 Faraday's Law

In this lab, Faraday's law is applied to determine the magnetic field in a permanent magnet's gap and the inside of a solenoid.

### 11.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

### 11.2 Introduction

Faraday's Law is the starting point in the study of electrodynamic fields. In this lab, Faraday's Law is used to obtain estimates of the magnetic field associated with a permanent magnet and a time-varying current.

The change in magnetic flux through a coil induces a current along the coil. The motion of charge along the coil is associated with an induced electric field that runs along the wire in the direction that positive charges are moving.

According to Faraday's Law, a voltage is induced by a changing magnetic flux. The polarity of this voltage is determined by Lenz's Law, which requires the current resulting from the induced voltage to run in a sense such that it generates a magnetic field that opposes the change in the original flux. In general, Faraday's Law can be written as

$$\mathcal{E}(t) = -\frac{d}{dt} \int_A \vec{B} \cdot d\vec{a}, \quad (1)$$

where the integral represents the flux,  $\Phi$ , of the magnetic field through the area of the loop,  $A$ . The derivative in front of the integral indicates that the total time rate of change of the magnetic flux must be taken. Note that for the cases considered here, the magnetic field  $\vec{B}$  is assumed to be uniform over the area of integration and also parallel to the area vector,  $d\vec{a}$ . In this case, the magnetic flux can simply be written as

$$\Phi = BA, \quad (2)$$

where  $A$  is the area of integration and  $B$  the magnitude of the magnetic field. It should be clear that the flux can change with time if the magnitude of the magnetic field, the area integrated over, or both change with time.

*A word on notation:*

For historical reasons, the voltage (work per unit charge) measured at the terminals of the coil is sometimes called electromotive force (EMF) and often denoted with  $\mathcal{E}(t)$ . Historically, the EMF has been associated with non-electrical sources of the electric field (like batteries).

So far, we have been using the word voltage to denote electric potential difference, more specifically, the difference between the electric potential at a given point and a reference point, called ground (zero potential). However, the electric potential is a function defined only for electrostatic fields, which is helpful in calculating the work per unit charge done by an electric field. In contrast, in the electrodynamic case, the electric field cannot be associated with an electric potential (make sure that you understand why). Nevertheless, we can still measure the work per unit charge done by the electric field, moving charges between two points. This work per unit charge is referred to as the voltage between the points.

In this lab, we keep the notation  $\mathcal{E}(t)$  for work per unit charge but refer to it simply as voltage. As usual, the units of choice to measure work per unit charge are Volts (V).

### 11.3 Relative Motion

In this part of the lab, Faraday's Law is used to determine the magnitude of the magnetic field  $B$ , in a permanent magnet. For this purpose, a coil of wire is placed between the poles of a large permanent magnet, which sets up a flux of  $B$  through the coil. Pulling the coil away from the poles reduces the number of magnetic field lines passing through it, so the flux through the coil is reduced. Far away from the magnet, the magnetic flux will be reduced to zero.

Note that in this case, neither  $B$  nor the area of the coil are changing in time, but the relative motion of the coil in a spatially-varying magnetic field produces a change in flux through the coil. Thus, according to Faraday's Law, we should observe a voltage generated across the terminals of the coil. The generated voltage can be measured by connecting the coil to a voltmeter. The voltage will vary with time depending both on the speed with which the coil is pulled out and the spatial variation of  $B$ , neither of which are known. However, the total change in flux is given by the difference between the final and initial magnetic fluxes.

If the magnetic field,  $B$  (between the poles of the magnet), is uniform over the area of the coil,  $A_c$ , then flux through a coil with  $N_c$  loops is simply  $\Phi = N_c A_c B$ . Hence, the change in the magnetic flux when pulling the coil away from the magnet is

$$\Delta\Phi = \Phi_{final} - \Phi_{initial} = -N_c A_c B. \quad (3)$$

This change in the flux can also be obtained by integrating Faraday's law (Equation 1) with respect to time,

$$\Delta\Phi = \int d\Phi = - \int \mathcal{E}(t) dt. \quad (4)$$

These results together imply that the magnetic field between the magnet's poles can be written as

$$B = \frac{1}{N_c A_c} \int \mathcal{E}(t) dt \quad (5)$$

To evaluate this integral the voltage must be known as a function of time.

## Procedure

A chart recorder, which records voltage as a function of time on a paper chart using an ink pen, will be used to generate plots of the coil's voltage as the coil is pulled away from the magnet.

Set the chart recorder speed to 60 cm/min and its range to 200 mV. Connect the ends of the coil to the recorder, place the coil between the poles of the magnet, wait for the recorder to settle down, and then pull the coil away from the magnet. Practice a few trials until you get a good plot.

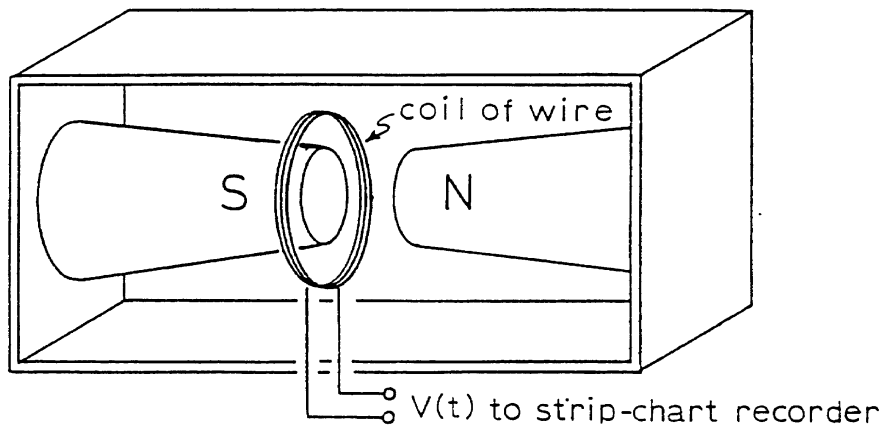


Figure 1: Diagram of setup for testing Faraday's Law.

## Data

The coil has 100 turns. Measure its diameter and record its value. Using the chart recorder speed and range setting, estimate the area under the  $\mathcal{E}(t)$  curve and write it next to the plot. Make photocopies for each member of your team.

## Analysis

Use your measurements to compute  $B$  for the permanent magnet.

### 11.4 Time Varying Magnetic Field

The magnetic field generated inside an air-core solenoid by a time varying current will be examined next. The results will be compared to the theoretical field predicted by Ampere's Law for an ideal solenoid with the same characteristics.

Using Ampere's Law, it can be shown that the magnitude of the magnetic field (in Tesla) inside an ideal solenoid of length  $\ell$  and  $N_s$  turns is given by

$$B = \mu_0 \left( \frac{N_s}{\ell} \right) I, \quad (6)$$

where  $I$  is the current flowing through the wire of the solenoid and  $\mu_0 = 4\pi \times 10^{-7} \text{NA}^{-2}$  is the permeability of free space.

Hence for a sinusoidal current of amplitude  $I_0$  and frequency  $f$ , the magnetic field is given by

$$B(t) = \mu_0 \left( \frac{N_s}{\ell} \right) I_0 \sin(2\pi ft). \quad (7)$$

For a less than ideal solenoid, the magnetic field is still proportional to the current, but the constant of proportionality may not be the one predicted by Ampere's Law. In this case, the magnetic field can be written as

$$B(t) = B_0 \sin(2\pi ft), \quad (8)$$

where  $f$  is the frequency of the input current.

This field can be sensed by a small coil inserted inside the solenoid, such that its area,  $A_p$ , is perpendicular to the axis of the solenoid. If the magnetic flux is uniform through the area of the pickup coil, the magnetic flux through the  $N_p$  loops of this coil is given by

$$\Phi(t) = (N_p A_p) B_0 \sin(2\pi ft), \quad (9)$$

According to Faraday's law, the induced voltage on the pickup coil is minus the time rate of change of this flux. Hence, it must be

$$\mathcal{E}(t) = -2\pi f N_p A_p B_0 \cos(2\pi ft), \quad (10)$$

$$= 2\pi f N_p A_p B_0 \sin(2\pi ft - \pi/2). \quad (11)$$

Hence, the amplitude of the induced voltage is given by

$$\mathcal{E}_0 = 2\pi f N_p A_p B_0. \quad (12)$$

## Procedure

**Inside the solenoid.** Set the signal generator to produce a sine wave with a frequency of 1 kHz and use the oscilloscope to set its amplitude to 10 V peak-to-peak. Connect the solenoid to the signal generator as shown in Figure 2. The  $100\ \Omega$  resistor is used to sense the current. Since it is connected in series with the solenoid, the same current flows through them. Measure the actual value of the resistor. Turn the signal generator on and measure the voltage across the resistor. (Ohm's Law is used to determine the current.)

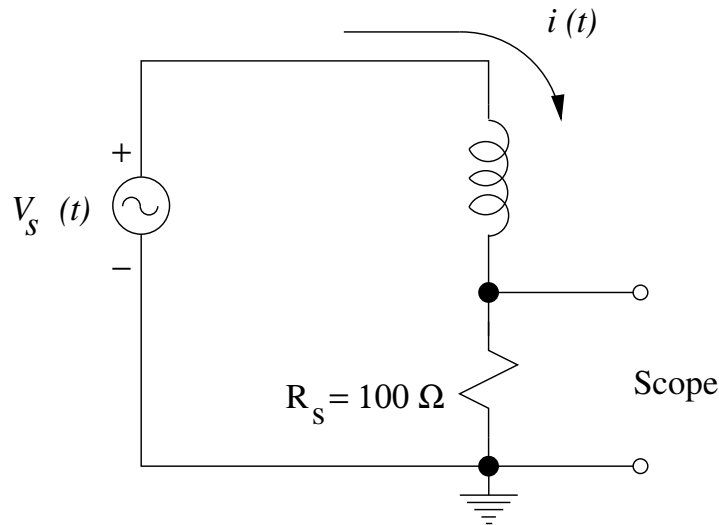


Figure 2: Circuit for supplying and measuring current to the solenoid.

Carefully place the pickup coil in the center of the solenoid and connect it to the oscilloscope. Measure the amplitude,  $\mathcal{E}_0$ , of the voltage induced in the coil by the solenoid's changing magnetic field.

**Outside the solenoid.** Once you finish your measurements, examine the surroundings of the solenoid. Start by placing the pickup coil on top of the solenoid at the center (on the strip of tape). Turn it straight up, with the coil close to the solenoid, so that its axis is perpendicular to that of the solenoid. Slowly drag the pickup coil to the end of the solenoid. You should see the signal rise. What does this mean? Sketch what you think the magnetic field lines should look like in and around the solenoid.

## Data

For the solenoid, record the number of loops, its length, the peak-to-peak voltage applied, and frequency of the signal applied. Also record the value of the resistor used to sense the current flowing through the solenoid and the peak-to-peak value of the voltage across the resistor.

The pickup coil has 50 turns of wire. Measure and record its diameter and the peak-to-peak value of the induced voltage in the pickup coil.

Make a sketch of the magnetic field in and around the actual solenoid.

### **Analysis**

Determine the current flowing through the solenoid. Use the value of this current and Ampere's Law to obtain the theoretical value of the amplitude of the magnetic field inside the ideal solenoid.

Use your measurements from the pickup coil and Faraday's Law to determine the amplitude of the magnetic field for the actual solenoid. Compare this value to the ideal value obtained from Ampere's Law.

### **11.5 Questions**

1. Derive Equation 6.
2. In both experiments, the voltage induced in the coil of wire could be connected to a resistor and would cause the resistor to heat up slightly. What is the source of this energy in each experiment?



## 12 Mutual Inductance

In this Lab, we investigate the Mutual Inductance between two coils in a concentric configuration.

### 12.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

### 12.2 Introduction

The coil's configuration is depicted in Figure 1.

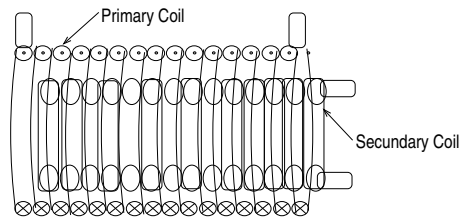


Figure 1: Primary and secondary coils in a concentric arrangement, with the secondary as the inner coil.

According to Ampere's Law, a current circulating in a solenoid produces a magnetic field along the solenoid. A schematic of this field is shown in Figure 2, and its magnitude is given by

$$B = \mu_0 \left( \frac{N_p}{L_p} \right) I_p, \quad (1)$$

where  $N_p$ ,  $L_p$ , and  $I_p$  are the number of turns, length, and current through the solenoid;  $\mu_0 = 4\pi \times 10^{-7} \text{ WbA}^{-1}\text{m}^{-1}$  is the permeability of free space. Once the secondary coil is inserted in the primary coil, as shown in Figure

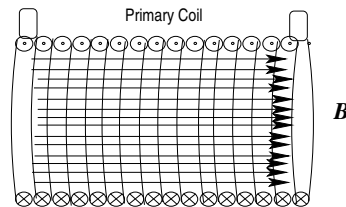


Figure 2: Magnetic Field along the axis of a solenoid.

1, the magnetic flux expected to go through each turn of the the secondary coil is simply

$$\phi_s = BA_s, \quad (2)$$

where  $A_s$  is the area of the secondary coil, and we have assumed that the magnetic field is uniform throughout the area of the primary coil. In this configuration, the coils' area is fixed, and there is no relative motion between them. According to Faraday's law for a static magnetic field in the primary coil, there should be no voltage difference between the terminals of the secondary coil. However, according to equations (1) and (2), a changing current in the primary coil produces a changing magnetic flux through the secondary coil, which in turn induces a voltage across each turn of the secondary coil. In this case, the voltage induced at the terminals of the secondary coils is

$$\varepsilon_s(t) = -N_s \frac{d\phi_s}{dt}, \quad (3)$$

where  $N_s$  is the number of loops in the secondary coil.

When the magnetic field produced by one coil is felt by another coil, we say that the coils are linked by the magnetic field. The big assumption is that for any pair of coils in any configuration, the magnetic flux passing through the secondary coil is proportional to the current,  $I_p$ , in the primary coil (the coil producing the magnetic field). Representing the magnetic flux per loop in the secondary coil by  $\phi_s$ , we could write the total flux as

$$N_s\phi_s = MI_p, \quad (4)$$

where  $N_s$  is the number of turns in the secondary coil, and  $M$  is a constant that only depends on the coils' configuration. The constant of proportionality  $M$  is called the Mutual Inductance of the coils.

It is worth noting that if the coils' roles are interchanged, but their configuration is the same, then equation (5) remains valid. The SI unit for  $M$  is the henry (H). From equation (4), it can be seen that  $1 \text{ H} = \text{WbA}^{-1}$ .

Combining equations (3) and (4), we obtain

$$\varepsilon_s(t) = -M \frac{dI_p}{dt}. \quad (5)$$

Assuming that the current through the primary can be represented as

$$I(t) = I_0 \sin(\omega t), \quad (6)$$

where  $I_0$  and  $\omega$  are its amplitude and angular frequency, then according to equation (5) the voltage across the secondary must be

$$\varepsilon_s(t) = MI_0\omega \cos(\omega t). \quad (7)$$

Calling  $\varepsilon_{s0}$  the amplitude of the voltage across the secondary, it follows that

$$\varepsilon_{s0} = M\omega I_0. \quad (8)$$

We use equation (8) to determine experimentally the Mutual Inductance, and compare our results with the theoretical prediction for  $M$ , which is obtained next.

For the configuration that is shown in Figure 1, the magnetic field in the primary is given by equation (1). Hence, the magnetic flux through the secondary coil is simply

$$\phi_s = \frac{\mu_0 \pi D_s^2 N_p}{4L_p} I_p, \quad (9)$$

where the  $D_s$  is the diameter of the secondary coil. It follows from equation (4) that for this configuration the Mutual Inductance is given by

$$M = \frac{\mu_0 \pi D_s^2 N_p}{4L_p} N_s \quad (10)$$

Note that equations (9) and (10) are valid for vacuum and to a good approximation for air, and any non-ferromagnetic material. However, the magnetic field and therefore the magnetic flux are significantly increased in the presence of ferromagnetic materials. In this case, we must replace  $\mu_0$ , with the actual permeability of the material.

### Procedure

To investigate the relationship between the current in the primary and the induced voltage in the secondary coil shown in Figure 1, We use a signal generator and an Oscilloscope. Ask your instructor if you have questions about the operation of these instruments.

The setup for this experiment is shown in Figure 3. Note that the primary coil and the 1 k $\Omega$  resistor are connected in series. Hence, according to Kirchhoff's law, the current going through the coil is the same as the current going through the resistor. The signal generator is used to apply a sinusoidal voltage of various amplitudes, but with the same frequency, to drive a sinusoidal current in the primary coil. The current in the primary is tracked by monitoring the voltage drop across the resistor in the circuit (recall that current and voltage across a resistor are related by Ohm's law); the voltage across the secondary coils is observed directly on the oscilloscope.

Obtain a resistor of about 1 k $\Omega$ ; use a DMM to determine the actual resistance and record its value. Install the resistor on the screw-down board.

Verify the calibration of the Oscilloscope and then use it to set the signal generator to produce a 1 kHz sinusoidal signal of 2 Vp-p.

Connect the primary coil, the 1 k $\Omega$  resistor, and the signal generator in series, as shown in Figure 3. Changing the amplitude of the input signal

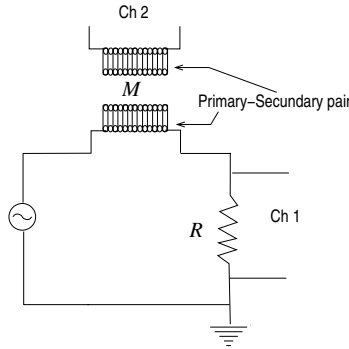


Figure 3: Diagram of the setup to examine the Mutual Inductance between coils; the number of turns of the secondary coil is  $N_s = 235$ ;  $R$  has a nominal value of  $1 \text{ k}\Omega$ ;  $M$  is the Mutual Inductance.

changes the current across the primary inductor. Use the Oscilloscope's Ch 1 to monitor this current by looking at the voltage drop across the resistor. Use Ch 2 to monitor the voltage across the secondary coil.

### air-core solenoid

The coils are made out of copper wire wind around tubes of insulating material; the tube's core is just air. Insert the secondary coil into the primary; monitor the voltage across the resistor on Ch 1 and the voltage across the secondary coil on Ch 2. Vary the voltage drop across the resistor from  $2 \text{ V p-p}$  to  $12 \text{ V}$  in increments of  $2 \text{ V p-p}$  and measure the voltage across the secondary coil; record your measurements.

### iron-core solenoid

Reduce the voltage drop across the resistor to about  $1 \text{ Vp-p}$ , and remove the secondary from the primary coil. Insert the iron bar into the core of the secondary. Reinstall the secondary (with the iron core) back into the primary coil. Adjust the voltage drop across the resistor to be  $1 \text{ Vp-p}$ ; proceed to increase the voltage drop across the resistor, and collect the same kind of data as in the case of the air-core solenoid.

### Data

Organize the data for each case in a table that includes headings and units. You should have at least six pairs of measurements per case.

### Analysis

Determine if your data supports equation (8). For each case (air and iron core), calculate the current in the primary coil, and make a plot of the voltage across the secondary vs. current in the primary; use a ruler to draw the best fit line for your data; write the equation for the line and determine its slope. Use your experimental values for the Mutual Inductance to determine the permeability for the air and iron cores; compare your results with the permeability of vacuum.

Combine all of your plots, including the fitted lines, in the same graph using the same scale. Mark the data for each case with a different symbol and add an appropriate legend. Include this graph in your report as part of the analysis.



## 13 Displacement Current

In this Lab, we investigate the displacement current through a Capacitor in an LR-circuit. We compare the displacement and conduction current in this circuit.

### 13.1 Pre-Lab

Read this lab and come to class with questions on anything you do not understand.

### 13.2 Introduction

For this Lab, we use the configuration depicted in Figure 1.

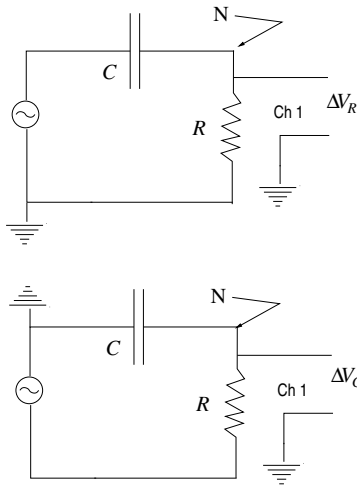


Figure 1: Capacitor in series with a resistor joined at node N, where we connect them to Ch 1. The top panel shows the resistor grounded, while in the bottom panel, the capacitor is grounded. Both configurations set the same conduction current through the resistor.

Connecting a DC power supply to the circuit will charge the capacitor and set its potential difference to the voltage of the power supply. For a brief moment, an electric charge will move from one plate of the capacitor to the other, but not across the gap of the capacitor. After the capacitor is charged, no more transport of charge will occur. At this point, the Electric field inside the capacitor is at its maximum value and will not change anymore.

Using an AC power supply changes the behavior of the circuit. The oscillating voltage at the plates of the capacitor sets an oscillating Electric

field between the plates, which in turns induces a Magnetic field inside the plates of the capacitor. Figure 2 depicts the situation at a given point in time. Notice the circular nature of the Magnetic field around the Electric

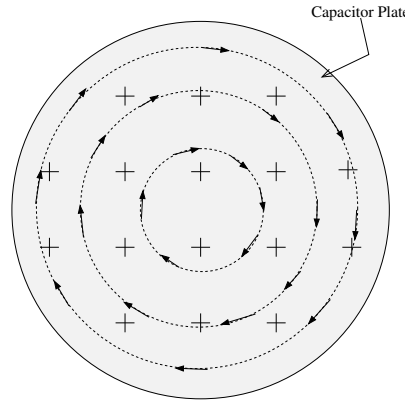


Figure 2: Electromagnetic fields inside a capacitor when the voltage across the capacitor changes as a function of time. The Crosses indicate an Electric field into the plane of the figure, while the arrows indicate the associated Magnetic field.

field. The Magnetic field behaves as if it were produced by a conduction current (charge moving) between the capacitor's plates.

The origin of the Magnetic field was explained by Maxwell <sup>1</sup>, who proposed that the changing electric Flux inside the capacitor is responsible for the emergence of the Magnetic field, just like a changing magnetic Flux inside a solenoid induces an Electric field. To account for this effect, Maxwell added an extra term into Ampere's law. He called it, the displacement current  $I_d$ .

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(I_c + I_d). \quad (1)$$

The first term on the right hand side of this extended Ampere's law is the standard conduction current, which is related to the motion of charge; the second term is the displacement current, which is given by

$$I_d(t) = \epsilon_0 \frac{d}{dt} \int_A \vec{E} \cdot d\vec{a}, \quad (2)$$

where  $A$  is the area of the capacitor's plates; the integral is the flux of Electric field across the area of the capacitor. The rate of change of this flux is what determines the magnitude of the Magnetic field in the gap of the capacitor. The direction of the Magnetic field is obtained by the right-hand rule (fingers curl in the direction of the Magnetic field, when the thumb points in the direction of the changing Electric field).

<sup>1</sup>James Clerk Maxwell (1831-1879). Scottish physicist, who unified in a single theory all electricity and magnetism phenomena

In terms of the Potential difference across the capacitor, the magnitude of the Electric field is

$$E = \frac{V_c(t)}{d}, \quad (3)$$

where  $d$  is the distance between the capacitor's plates; the direction of the Electric field is from the higher to the lower potential and it alternates with the same frequency as  $V_c(t)$ .

Assuming that the Electric field is uniform across the area of the capacitor simplifies the integral in equation (2). In this case, the displacement current is given by

$$I_d(t) = \epsilon_0(A/d) \frac{d}{dt} V_c(t); \quad (4)$$

Since the capacitance for this geometry (parallel plate capacitor) is given by  $C = \epsilon_0(A/d)$ , the displacement current can be written as

$$I_d(t) = C \frac{d}{dt} V_c(t); \quad (5)$$

Applying the sinusoidal voltage

$$V_s(t) = V_{s0} \sin(\omega t) \quad (6)$$

to the circuit of Figure 1, sets up a sinusoidal voltage across the resistor and capacitor. These voltages have the same frequency as the input voltage, but in general, will have different amplitudes and phases. The voltage drop across the capacitor can be written as

$$V_c(t) = V_{c0} \sin(\omega t + \phi_c), \quad (7)$$

where  $V_{c0}$ ,  $\omega$ , and  $\phi_c$  are the amplitude, angular frequency and phase constant, respectively. Similarly for the resistor, the voltage drop can be written as

$$V_R(t) = V_{R0} \sin(\omega t + \phi_R). \quad (8)$$

Substituting equation (7) into (5), results in

$$I_d(t) = C\omega V_{c0} \cos(\omega t + \phi_c). \quad (9)$$

Hence, the amplitude of the displacement current is

$$I_{d0} = C\omega V_{c0} \quad (10)$$

On the other hand, the amplitude of the conduction current through the resistor is, by Ohm's law,

$$I_{c0} = V_{R0}/R. \quad (11)$$

According to Kirchhoff's Node Law, the current through elements connected in series must be the same. In this Lab, We test whether the amplitudes of the conduction current, through the resistor, and the displacement current, in the capacitor, are the same.

## Procedure

We will use a signal generator and an Oscilloscope to perform the experiment. Ask your instructor if you have questions about the operation of these instruments. The setup for this experiment is shown in Figure 1.

To measure the conduction and displacement currents start by procuring a resistor with a resistance of about  $30\text{ k}\Omega$ ; determine its actual value using a multimeter. Next set the gap of the capacitor to about half a centimeter and measure its capacitance using an LCR meter.

Verify the calibration of the Oscilloscope and then use it to set the signal generator to produce a sinusoidal signal with  $6\text{ V}$  peak-to-peak and a frequency of  $30\text{ kHz}$ . Connect the signal generator to Ch 1 and measure the actual amplitude and period of the input signal. After setting up the signal generator, turn it off and disconnect it from the Oscilloscope.

On the screw-down board, connect the capacitor in series with a resistor and signal generator using banana cables.

To measure the voltage drop across the resistor make sure that the AC source is connected as shown on the Top panel of Figure 1. Ensure that Ch 1 of the oscilloscope is connected to Node N; in this configuration, Ch 1 will show the Voltage drop across the resistor; turn on the signal generator and measure the amplitude of the voltage drop across the resistor.

According to Kirchhoff's loop law, we can find the voltage across the capacitor by taking the difference between the input voltage and the drop across the resistor. However, this difference depends not only on the amplitudes, but also on the phases of these voltages— see equations (6), (7), and (8). For this reason, we use the configuration shown in the Bottom panel of Figure 1, which allows for a direct measurement of the voltage drop across the capacitor. To set this configuration without modifying the original settings, only reverse the cables at the signal generator.

After measuring the voltage drop across the resistor, turn off the signal generator and reverse its connection to the circuit (be careful and do not change the amplitude and frequency); once the connection is reversed, Ch 1 will show the voltage drop across the capacitor; turn on the signal generator and measure the amplitude of the voltage drop across the capacitor.

Repeat your measurements for the rest of the input voltages given by your instructor. Note that instead of directly measuring the amplitude, it may be easier and more accurate to measure the peak-to-peak range and obtain the amplitude from those measurements.

### **Data**

Record the actual resistance and capacitance of the components in your circuit. For each input voltage, you should have recorded the period of the input signal and its amplitude; also the amplitudes of the voltage drop across the resistor and capacitor; Organize your data in a table that includes headings and units.

### **Analysis**

To compare the amplitude of the conduction and displacement currents, make a plot of one against the other. If the currents are the same, your plot should resemble a straight line. What should be the value of the slope?

Compare your slope to the expected value.

