PHYSICS 1310 LAB MANUAL

Department of Physics
New Mexico Institute of Mining and Technology

Fall 2020

Preface

Our freshman laboratory program aspires to provide students with engaging and rewarding interactions with the physics of the world around them. The weekly laboratories address a cross-section of the concepts introduced in the lecture part of the course. The *Physics Laboratory Manual* is the handbook for the class and introduces the student to the techniques of obtaining and analyzing experimental observations using diverse methods and equipment. The sessions are led by a Teaching Assistant (TA), who is usually a physics graduate student.

Your TA will provide you with a syllabus during the first lab session that will outline the requirements for each lab and its associated laboratory report. In general, students are asked to report their activities and results clearly and concisely in formats ranging from "worksheets" to a formal lab report write-up. The time spent outside of lab on preparation, data analysis, and presentation will vary somewhat on the lab and on the TA, and especially on how much work is done while in the lab. An average time of no more than 3 hours of outside work per lab is encouraged.

Some guidelines to remember throughout the semester:

1. Make sure to always get your raw data sheet signed and/or initialed by your TA prior to leaving the lab session.
2. Punctuality in attendance and completion of assignments.
3. Rotation of individual roles in a team effort to ensure everyone's participation in the exercises.
4. Approaching all issues with professional courtesy and respecting the efforts, opinions, and property of others.
5. **Disruptive behavior**, including horseplay and reckless use of equipment, will NOT be tolerated!

In an effort to provide the most interesting and interactive physics labs possible, the department of physics at New Mexico Institute of Mining and Technology undertook a significant revision of its lab course manual (Physics 121) during the 2013 – 2014 academic year. This involved major modifications and improvements to already existing labs, and building labs combining ideas from several different sources and universities. Some of the worksheets used for labs are adapted from those written by graduate students Satomi Sugaya and Sam Collopy.

Of course, nothing is ever done in isolation, and our efforts relied heavily on the work of those who came before, most notably the original author of a major portion of the labs found within this manual. Also, Dr. Barry Sabol provided useful, additional material and information for the labs over the years, which directly led to several of the modifications made. We'd like to also thank current and previous NM Tech physics graduate student instructors and physics professors for their efforts on behalf of this course, including informative and critical feedback.

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Tina Güth

June 2014

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Lab 1

Vector Addition of Forces

In this lab, we introduce vectors and demonstrate vector addition using a familiar vector quantity –force. We do this experimentally by using a force table, which allows three separate forces to be in *static equilibrium*. We also balance the forces graphically and mathematically.

1.0.1 Background

Vectors are quantities that have both a magnitude and a direction. You can think of them as packets of information. They pack a bit more information than scalars, which only contain magnitude information.

Describing a Force, requires both, the specification of its magnitude and the direction along which it is being applied. Hence, forces are described as vector quantities. In this lab manual, we write these quantities with an arrow overhead to indicate that such quantity has not only magnitude but directionality, e.g., $\vec{F}$. When we refer to only the magnitude of a vector we, usually skip the arrow and write F. However, sometimes we write $|\vec{F}|$ to emphasize that we are only talking about the magnitude of the vector $\vec{F}$.

Specifying direction can be done in various ways. Figure (1.1) shows a simple example, where the direction is specified by the angle, θ, measured counterclockwise from the x axis. Another way to specify the same direction is by giving the angle measured clockwise from the y axis.
Still another way to specify the direction of a vector is by providing the slope of the line along the vector, which is equivalent to specify the tangent of θ. For example, calling F_x the leg along the x axis and F_y the leg along the y axis, we can write

$$\tan(\theta) = \frac{F_y}{F_x}. \tag{1.1}$$

Note that by specifying those legs, we are implicitly specifying the magnitude of the vector, that is

$$F = \sqrt{F_x^2 + F_y^2}, \tag{1.2}$$

where we have use the Pythagorean theorem. We normally refer to these legs as the components of the vector.

Since forces are vector quantities, when more than one force acts on a body, the rules of vector addition must be used to determine the resultant force. These rules are slightly more complicated than the addition of scalars due to the directionality information, which typically requires more than one dimension. In this lab, we limit ourselves to vectors in two dimensions.

Experiment

We use a force table to find the force that balances the two that are applied. A force table is a circular table that we use to apply forces of different magnitude and direction to a small ring in the center. Three strings are attached to the ring, run over pulleys on the edge of the table, and terminate on mass hangers, which hold known masses. The pulleys transfer the vertically directed gravitational force of the masses to a horizontal tension force on the ring. The magnitudes of the forces can be adjusted by changing the masses, and the directions can be adjusted by moving the pulleys around the table (marked from 0° to 360°). Thus, when we start with two fixed forces, we can adjust the third to balance them and create static equilibrium as seen in Figure (1.2).

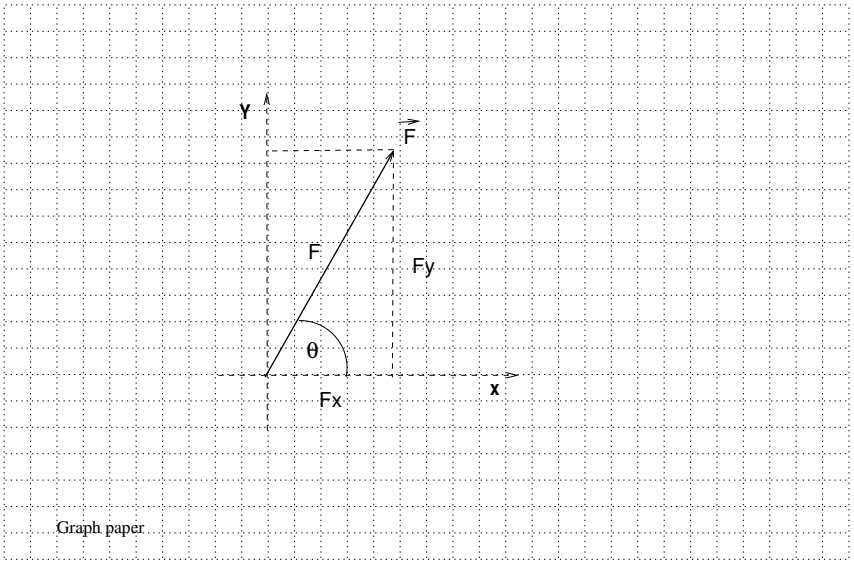

Figure 1.1: Vector: magnitude and direction

Graphically

Referring to Figure (1.3), the resultant force ($\vec{R}$) due to vectors $\vec{A}$, and $\vec{B}$ is just the vector addition of the two vectors. To perform this sum graphically, start by drawing vector $\vec{A}$ with its tail at the origin of coordinates. Next draw vector $\vec{B}$ with is tail at the head of vector $\vec{A}$. Both vectors must be drawn to scale. The resultant vector goes from the tail of vector $\vec{A}$ to the head of vector $\vec{B}$ (tail-to-head). The balancing vector $\vec{C}$ must be of the same magnitude as $\vec{R}$ but in the opposite direction (it goes head-to-tail). Note that vectors may be drawn anywhere in the plane as long as they maintain their magnitude and orientation.

Mathematically

In this section, we show how to reduce the two dimension problem of adding vectors to several one dimensional problems, which we know how to solve. The resultant force $\vec{R}$ is found from two applied forces $\vec{A}$ and $\vec{B}$ according to the following process. First, break $\vec{A}$ and $\vec{B}$ into their Cartesian components:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors specifying direction along the x and y axes, respectively, and their coefficients are the corresponding magnitudes. In this way, we reduce the problem to two one-dimensional problems: one along the x-direction and one along the y-direction, where we only add scalars using the usual rules of arithmetic.

Adding matching components arithmetically:

$$\vec{R} = \vec{A} + \vec{B}$$

$$\vec{R} = A_x\hat{\imath} + A_y\hat{\jmath} + B_x\hat{\imath} + B_y\hat{\jmath}$$

$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$

Once we have the components of the resultant vector, we could find its magnitude and the angle it makes with the x axis.

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}, \tag{1.3}$$

$$\tan(\theta) = \frac{A_y + B_y}{A_x + B_x}. \tag{1.4}$$

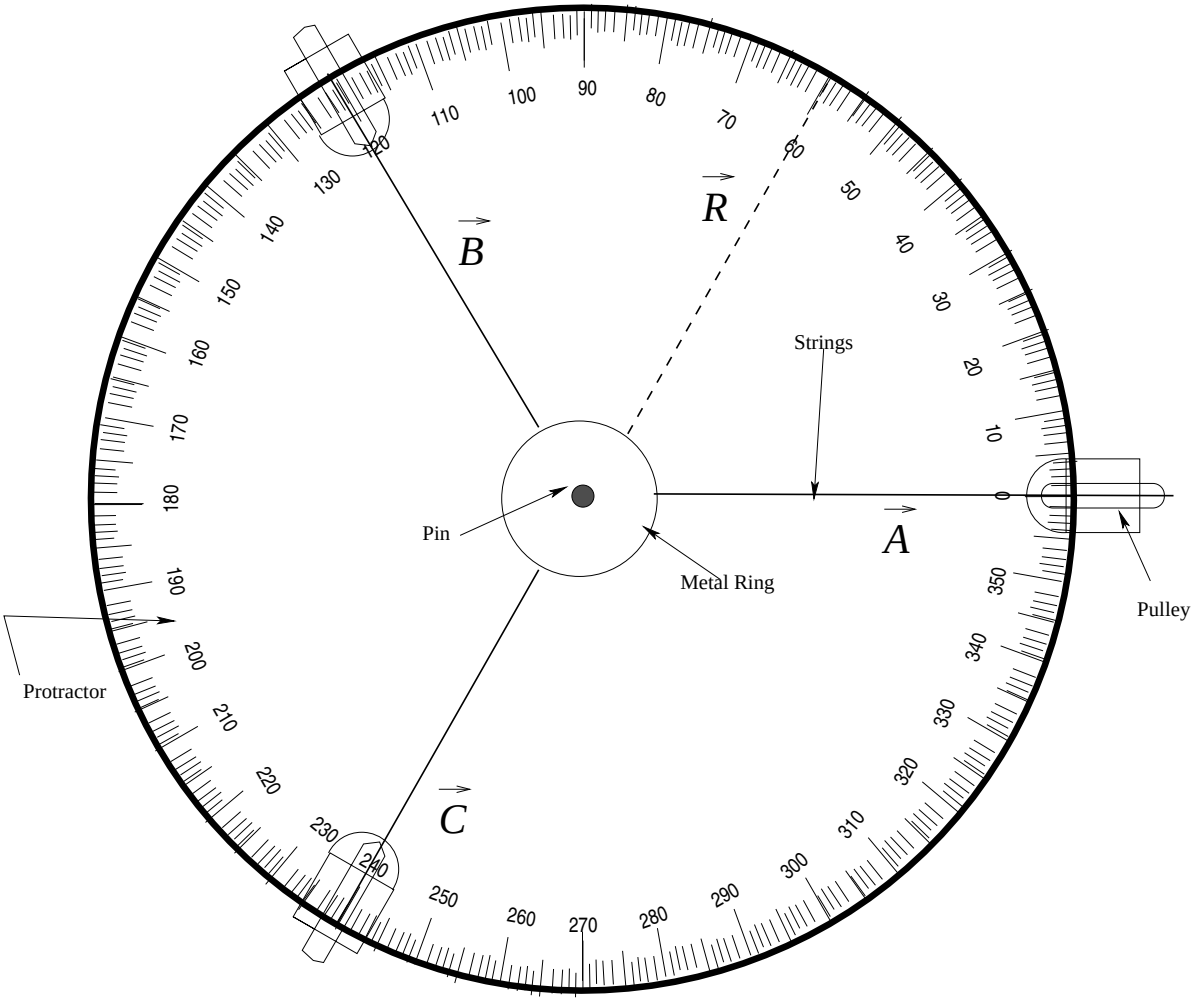

Figure 1.2: Schematic demonstrating a force table in static equilibrium. Note that the pin should be at the center of the metal ring; vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{R}$ point away from the center of the table.

In this lab, we are examining static equilibrium, which means that the net force (that is, the total sum of all forces) is zero. The net force is simply

$$\vec{R} = \vec{A} + \vec{B}.$$

To balance this force, we must apply a force of equal magnitude, but in the opposite direction. That is

$$\vec{C} = -\vec{R},$$

so that

$$\vec{A} + \vec{B} + \vec{C} = 0.$$

When the force is given as a magnitude and an angle of direction (as is the case here), use trigonometry to break it into its Cartesian components.

Example

Referring to Figure 1.2, Force $\vec{B}$ is given as 10 N at 120° from the positive x-axis. Its x component is pointing in the negative direction ($-\hat{\imath}$), while its y component points in the positive direction ($\hat{\jmath}$):

$$B_x = |\vec{B}| \cos(\theta),$$

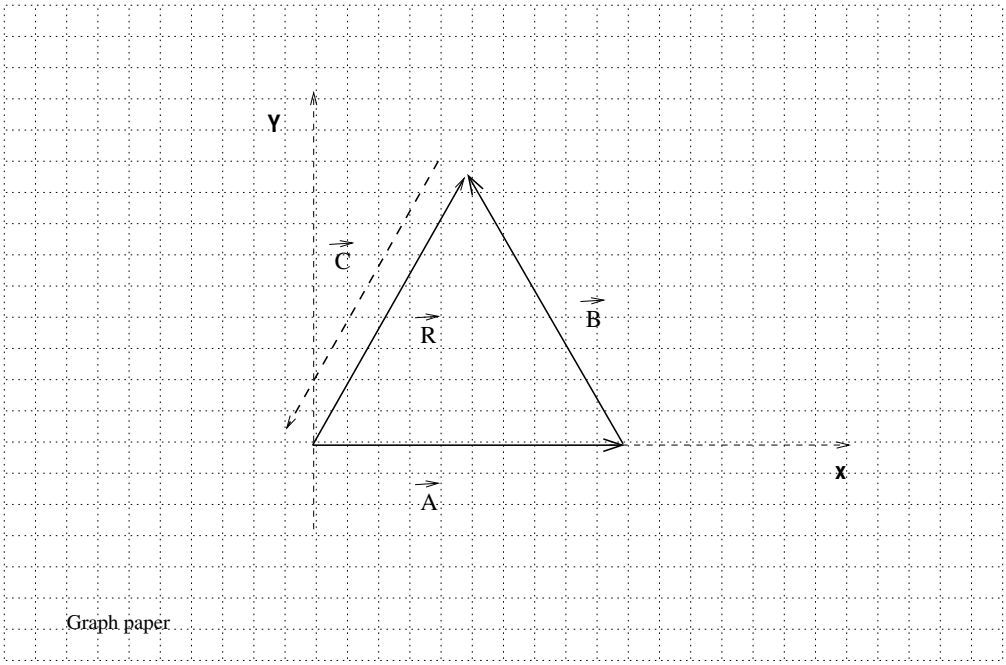

Figure 1.3: Graphic addition of vectors $\vec{A}$ and $\vec{B}$. Both vectors must be drwan to scale and maintain original orientation. The resultant vector ($\vec{R}$ = $\vec{A}$ + $\vec{B}$) is shown as an arrow from the tail of $\vec{A}$ to the head of $\vec{B}$. A balancing vector $\vec{C}$ to counteract $\vec{R}$ is shown as a dashed arrow with the same magnitude of $\vec{R}$, but in the opposite direction.

$$B_x = (10N)\cos(120^\circ) = -5N.$$
$$B_y = |\vec{B}|\sin(\theta),$$
$$B_y = (10N)\sin(120^\circ) = 8.66N.$$

Force $\vec{A}$ is given as 10 N at 0° from the positive x-axis. Hence,

$$A_x = (10N)\cos(0^\circ) = 10N,$$
$$A_y = (10N)\sin(0^\circ) = 0N.$$

Adding these two vectors results in

$$\vec{R} = \vec{A} + \vec{B},$$
$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath},$$
$$\vec{R} = [(10 - 5)\hat{\imath} + (0 + 8.66)\hat{\jmath}]N,$$
$$\vec{R} = [5\hat{\imath} + 8.66\hat{\jmath}]N$$

If we have the x and y components (i.e. $\mathrm{R_x}$ and $\mathrm{R_y}$) of a vector, the magnitude can be determined as follows

$$|\vec{R}| = (R_x^2 + R_y^2)^{1/2} = ((5)^2 + (8.66))^2)^{1/2}N = 10N\ .$$

To determine the angle, we first determine the quadrant the resultant vector falls into and then use the magnitudes of the components to obtain the angle with the x-axis. In the current case, the resultant vector falls into the 1st quadrant ($R_x = 5N$, $R_y = 8.66N$) and the angle with the x-axis is given by

$$\theta = \arctan(|R_y|/|R_x|) = \arctan(8.66N/5N) = 60^\circ.$$

To balance the resultant force, we must add a counterweight on the Force table of the same magnitude, but in the opposite direction. This balancing force is represented by vector $\vec{C}$.

1.0.2 Procedure

The following steps provide direction for the vector addition of three forces using the experimental procedure. There are three different cases in all, as listed below.

Case 1:

$$\vec{A} = 0.98N \text{ at } 0^\circ$$
$$\vec{B} = 0.686N \text{ at } 60^\circ$$

Case 2:

$$\vec{A} = 0.98N \text{ at } 90^\circ$$
$$\vec{B} = 0.98N \text{ at } 315^\circ$$

Case 3:

$$\vec{A} = 1.372N \text{ at } 180^\circ$$
$$\vec{B} = 0.98N \text{ at } 150^\circ$$

Experimentally

1. Calculate and record the masses needed to produce the forces given in each case above. Use Newton's second law, $\vec{F} = m\vec{a}$, assuming that the magnitude of $\vec{a}$ is equal to 9.8 m/s^2. Pay careful attention to the units!
2. Attach the calculated masses to their corresponding strings and move them to the required angular positions. Keep in mind that the hanger itself has a mass of 50 g and that the masses calculated above are the total required.
3. Vary the mass and angle of the third force until equilibrium is reached. That is, adjust both the mass on the hanger of the third string and its angular position until the metal ring is suspended at rest and centered on the central pin of the table. You may need to flick the ring lightly as frictional forces can sometimes keep it from its true equilibrium position.
4. Record the angle and total mass of the third string in your raw data sheet. Repeat for the other two cases.

1.0.3 Analysis

Experimentally

i. For each case, use the mass determined in the **Procedure Section** to calculate the magnitude of the balancing force. Show your work.

Graphically

i. Draw and label coordinate axes with a ruler on a piece of graph paper (graph paper is provided at the end of the *lab manual*). To ensure accurate results, use a protractor and ruler. Note that 0° defines the x-axis, while 90° the y-axis.

ii. Draw, to scale, the forces given for each case. Follow the method illustrated in the **Background Section**. Be sure to label your forces and provide the scale used (e.g., 1 N = 5 cm). Each case has to be drawn on a separate set of coordinate axes.

iii. Determine the balancing force from the graph by measuring the length of the resultant vector. Use the scale you decided on in the previous step and convert the length into Newtons (N). Also, determine the resultant angle from the graph you drew.

iv. For the angle, consider the quadrant of the resultant vector. Make sure to record the angle value for each case.

v. Attach all three graphs, including the scale used, with your lab report.

Mathematically

i. Break the forces given in each case into its Cartesian components. Follow the method outlined in the **Background Section** and show all of your work.

ii. Calculate the magnitude and direction angle of the resultant force. Be careful when determining the angle by considering the quadrant of the resultant vector.

Lab 2

Kinematics

2.1 Background

In this lab, we study the description of a particle's motion. The objective is to recognize constant velocity and various types of accelerated motion from looking at the track left by a moving particle. In particular, we develop a simple technique to obtain velocity and acceleration information from measurements of time and position. This technique will serve us well later in the semester when we use it to study conservation of energy and momentum. Nevertheless, the focus of this lab is to recognize that even when experimentally we can only measure average velocities, we can still recognize the influence of external forces acting upon an object just by tracking its position with time.

During the analysis part of this lab, you will be asked to compare some measurements. In physics, this task has a particular meaning, which we now describe. We can compare measurements by calculating their relative percentage difference with respect to some arbitrary reference (typically taken as one of the measurements). For instance, if we measure the speed of an object at the beginning and end of an experiment, we can compare their values by taking their difference and then compare this difference to the initial value of the speed. In this case, the relative percentage difference is given by

$$\frac{v_f - v_i}{v_i} \times 100 \tag{2.1}$$

Note that by dividing the difference by one of the two quantities, we eliminate the units and therefore the scale. You will also be required to decompose displacement vectors into components. Figure (2.1) shows an example of how to break vectors into their Cartesian components. Ask your Instructor, if you have questions about how to perform these operations.

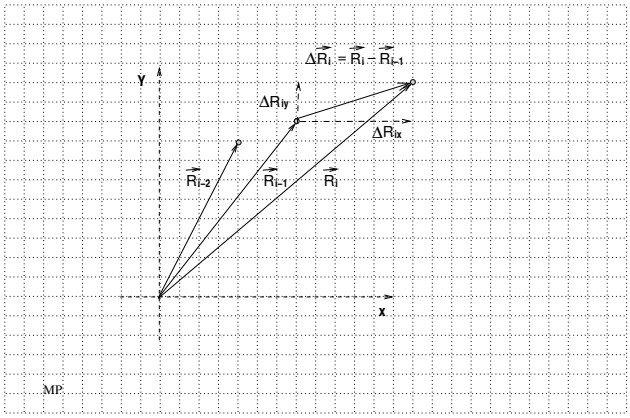

Figure 2.1: Vector Components. $\vec{R}_i$ indicates the i^{th} position vector, $\Delta\vec{R}_i$ indicates the displacement vector from $\vec{R}_{i-1}$ to $\vec{R}_i$; The components of $\Delta\vec{R}_i$ are also indicated; note that the position vectors are drawn at the actual locations of the object, we can measure the components of ΔR_i directly on the graph.

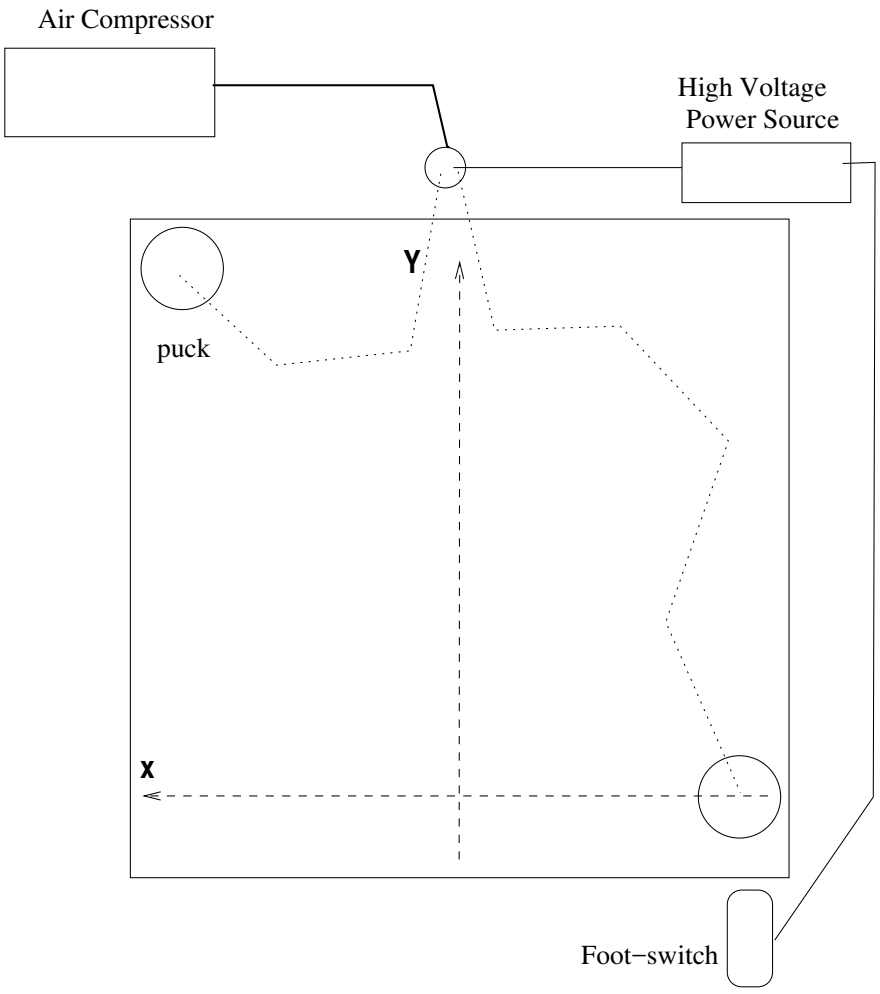

Figure 2.2: Schematic of the Air Table.

2.2 Procedure

The following steps provide direction to set up the equipment, perform the experiment, and prepare the data for analysis.

Equipment

Note: The air table used in this experiment uses high voltage to produce sparks. The current, on the other hand, is very small, so in principle, the table is not a life-threatening device. However, the high voltage can still sting you very much. **Keep your hands off the table when the electrical circuit is activated.**

Figure (2.2) shows the setup for this experiment. The air table has a glass surface, which we cover with a piece of special carbon paper that allows electricity to flow between the two pucks sitting on the table and connected to a power supply (sparking source). A sheet of Manila paper is then placed on top of the carbon paper, under the pucks. Every time there is a spark generated, there will be a mark left by the carbon on the Manila paper, at the location of each puck. As the puck moves, it leaves a trace which we refer to as the track or the graph. Note that while both pucks are required to be on the air table (to close the electrical circuit), we keep one at a corner and only track the motion of the other.

The friction between the pucks and the table is minimized by a flow of air running from a compressor to each puck.

The compressor provides air for the two tables in the lab.

For this experiment, we set the spark frequency to 20 Hz (20 sparks per second). This frequency is merely the inverse of the sparking period (the time interval between each spark). That is, one spark every 1/20th of a second (T = 1/f). The period between sparks is constant during the experiment, so we can always determine how much time has passed between any two marks left by the puck.

The electrical circuit is activated by the foot-switch on the floor by the table. It is recommended that the person who sets the pucks in motion is the same person that activates the circuit.

The following experiments can be done in any order, but it is recommended that all teams finish experiment one, before moving on to experiments two and three.

2.2.1 Experiment 1

1. Make sure that the air table is leveled;
2. Position one puck in one corner and the other near the left or right edge; this point is the starting position.
3. Practice setting the puck in motion, so it travels along a right-to-left direction.
4. To record the track of the puck, set the puck in motion as practiced and quickly activate the spark timer using the foot pedal. Be sure to turn off the spark timer as soon as the puck collides with the other edge of the air table.
5. At the end of the run, remove the spark sheet from the air table. **Make sure to mark the starting point on your spark sheet.** Confirm your result with your Instructor before moving on.

2.2.2 Experiment 2

1. Incline the air table by a about 5 degrees, with the highest end close to you.
2. Make sure that the air table is leveled along the left-right direction.
3. Position the puck near the highest side of the table and practice letting it go without pushing it.
4. Record the track as in experiment 1 and confirm the results with your Instructor.

2.2.3 Experiment 3

1. Keep the same inclination of the table as in experiment 2.
2. Make sure the manilla paper is well aligned with the sides of the air table, so that the x and y directions shown in Figure 2.2 are parallel to the table's sides.
3. Practice sending off the puck as in experiment 1 –with a very gentle push along the right-to-left direction.
4. Record the track as in experiment 1 and confirm the results.

2.2.4 Measurements

1. For each of the graphs, label the first dot as zero and index the rest of the dots according to your Instructor directions. Check with your Instructor if you are unclear about how to start the sequencing and labeling.
2. Draw displacement vectors between successive indexed dots.
3. For experiments one and two, measure the distances between successive indexed dots. Record these distances as Δx, for experiment 1, and Δy for experiment two, in a table with their corresponding index. These values are used in the **Analysis Section** to calculate the puck velocity along each direction.

4. For experiment three, break the displacement vectors into x and y components; measured the magnitudes of this components, and record them as Δx and Δy in your table.

2.3 Analysis

The first two experiments are performed, so the motion is one dimensional, while the third is two dimensional. The vector nature of the velocity should be clear by drawing arrows to represent the displacement vectors on the graphs. Recall that the velocity vector is the time rate of change of the position vector,$\vec{r}$. That is,

$$\vec{V} = \frac{d\vec{r}}{dt} \approx \frac{\Delta\vec{r}}{\Delta t}. \tag{2.2}$$

Writing this equation in terms of Cartesian components, we have

$$V_x = \frac{dx}{dt} \approx \frac{\Delta x}{\Delta t} \tag{2.3}$$

$$V_y = \frac{dy}{dt} \approx \frac{\Delta y}{\Delta t} \tag{2.4}$$

$$\tag{2.5}$$

2.3.1 Velocity

i. Draw arrows connecting the indexed dots on the track to illustrate the displacement vector, which coincides with the direction of the velocity;

ii. For each experiment, use the components of the displacement vector and the time between indexed dots to calculate the components of the average velocities.

2.3.2 Acceleration

To get insight on how the velocity changes, we use a graphical representation of the data. Since we control the time at which the measurements of the velocity are taken (we set the timer frequency), time is the independent variable while the velocity is the dependent one. Following the convention, the independent variable is plotted on the horizontal axis and the dependent variable on the vertical axis.

i. For each experiment, make a plot of the velocity components against time; your data should resemble a straight line. Using a ruler, draw a line that is as close as possible to all your data points. Beware of obvious outliers and do not force your line to accommodate them. The line that best represents your data points (excluding outliers) is called the best-fit line. Consult with your instructor to see if your fitted lines are acceptable.

ii. For each experiment, determine the slope of the best-fit line to get the corresponding acceleration. As with any straight line, the best-fit line is characterized by two parameters: its slope and its y-intercept. Its equation is simply

$$y = mx + b, \tag{2.6}$$

where x is the independent variable, m the slope, and b the y-intercept; the slope is given by

$$m = \frac{\Delta y}{\Delta x}. \tag{2.7}$$

In our case, x is time, and y is the velocity. Hence, the slope of the best-fit line is the acceleration, a. By the way, when calculating the the slope of the best-fit line, use points directly off the line and not the data points.

iii. For experiments one and three, compare the acceleration along the x axis. Finally, compare the acceleration along the y axis between experiments two and three.

Note: follow your instructor's directions concerning the submission of your spark sheet with your lab report.

Lab 3

Projectile Motion

3.1 Background

In this lab, we study a practical application of the vector equations of motion – projectile motion. Using a spring-loading launcher, we measure the range of projectiles for different launch configurations and compare these values with those predicted by theory.

Projectiles near the earth's surface trace out parabolas in their motions because gravity is the only force acting on them (ignoring air friction), and it acts only in the downward vertical direction. Therefore, when considering the vector equation of motion for a projectile, we break it into its Cartesian components and see that the horizontal velocity remains constant while the vertical velocity changes under the influence of gravity.
If we let the +x direction be horizontally to the right and the +y direction to be vertically up, then Figure (3.1) shows the velocity vector of a projectile that moves in a parabolic path.

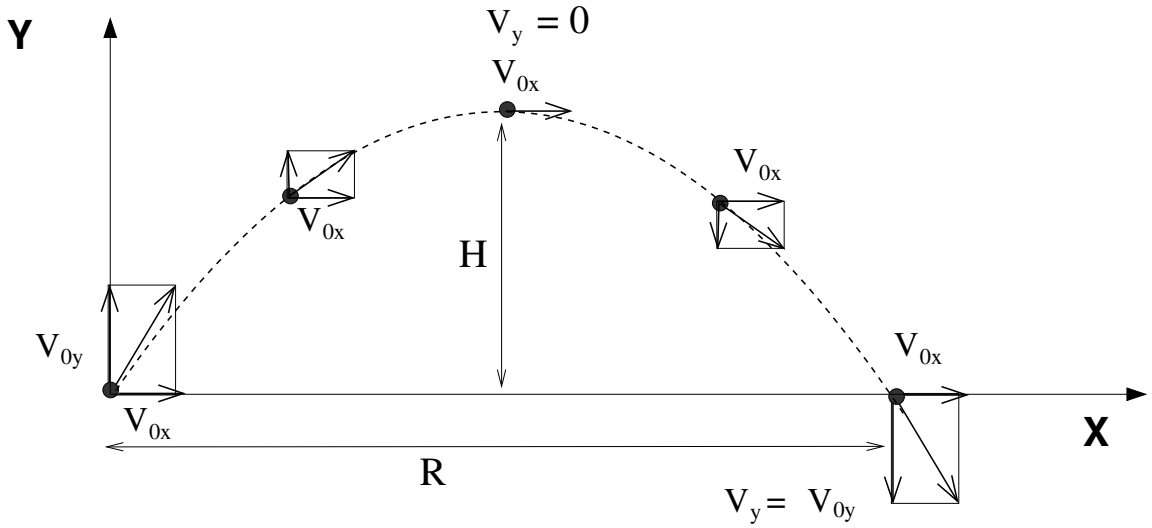

Figure 3.1: Schematic showing the x and y components of a velocity vector of a projectile moving along a parabolic path; R is the range of the projectile and H is its maximum height.

For constant gravitational acceleration pointing in the negative direction, the vector equation for velocity as a function of time is

$$\vec{v}(t) = \vec{v}_0 - gt\hat{j}, \tag{3.1}$$

which can be separated in Cartesian components as

$$v_x(t)\hat{\imath} + v_y(t)\hat{\jmath} = (v_{0x}\hat{\imath} + v_{0y}\hat{\jmath}) - gt\hat{\jmath} \tag{3.2}$$

or

$$v_x(t)\hat{\imath} + v_y(t)\hat{\jmath} = v_{0x}\hat{\imath} + (v_{0y} - gt)\hat{\jmath}. \tag{3.3}$$

We can group matching components and write this equation as two separate ones:

$$v_x(t) = v_{0x} \tag{3.4}$$

$$v_y(t) = v_{0y} - gt \tag{3.5}$$

From these expressions, we clearly see that the horizontal velocity is constant while the vertical changes with time.

We do the same to the equation expressing the projectile position as a function of time, given as

$$\vec{R}(t) = \vec{R}_0 + \vec{v}_0 t - \frac{1}{2}gt^2\hat{\jmath}, \tag{3.6}$$

which has the resulting two component equations

$$x(t) = x_0 + v_{0x}t, \tag{3.7}$$

and

$$y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2. \tag{3.8}$$

Notice that the x-position changes linearly in time because the x-velocity is constant while the y-position is quadratic in time. These are the two equations we apply to find the theoretical ranges in the projectile experiment.

The experiment in this lab involves a launcher that shoots projectiles. It has three different spring settings (short, medium, and long ranges), each one having its own muzzle speed, $\vec{v}_0$. The components of the velocity v_0 depend on the angle θ of the launcher with respect to horizontal. Hence, $v_{0x} = v_0\cos(\theta)$ and $v_{0y} = v_0\sin(\theta)$ as seen in Figure (3.2).

3.2 Procedure

This experiment involves several parts. The first is to measure the muzzle velocities of the launcher. The second is to perform two different launch experiments, each one with different settings for the launcher. And the final part is to find the angle for which the launcher shoots the projectile over the maximum range. See Figure (3.2) for the parameters of the experimental setup.

You must wear goggles at all times to protect your eyes!

3.2.1 Muzzle Velocity

1. Note that the photogate timer is already situated at the mouth of the launcher. The photogate timer consists of a main gate and an accessory gate that both emit an infrared beam. As the projectile interrupts the beam of the main gate, the timer is started and stops when the projectile interrupts the beam of the accessory gate.
2. Measure and record the distance between the main and accessory gates in your raw data sheet.
3. Set the photogate timer to 0.1 ms resolution and to PULSE mode. Make sure that the launcher is slightly tilted so that the ball is sitting against the launchpad in the nozzle for each range setting. Use the angle indicator on the side of the launcher to set a tilt of $\sim 1^\circ$, if not already done so.

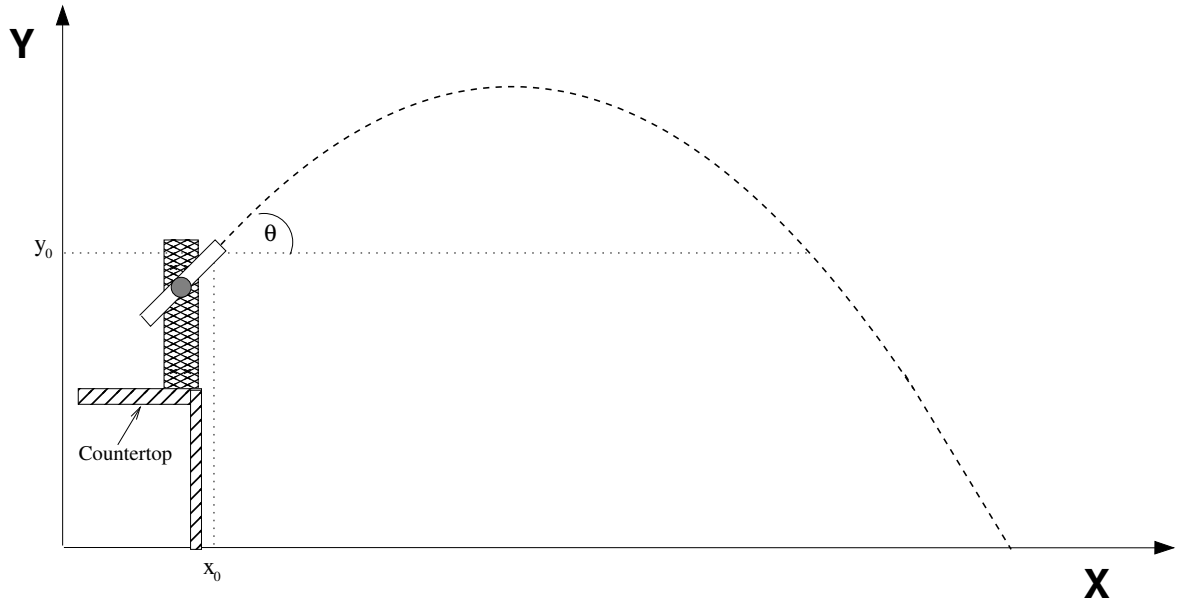

Figure 3.2: Diagram showing the parameters of the experiment setup. Note that the impact point is on the floor.

4. Set the launcher to **medium** range. Use the black rod provided to push the launchpad to the correct setting. Carefully insert the ball since some launchers are easily triggered!
5. Launch a projectile for three separate trials and record each displayed time in your raw data sheet. Be careful not to move the launcher and photogate during the trials.
6. Repeat for the launcher's **long** range setting.

3.2.2 Launch Experiments

1. Remove the photogate by loosen the bottom screw and sliding it off.
2. Set the launcher's parameters according to your TA's instructions. That is, set its angle θ, the range (either medium or long), and the impact height y (the height of the target area).
3. After setting the launcher to the required angle, measure and record the launch height, y_0, which is taken at the crosshair indicator on the side of the launcher mouth.
4. Sketch the setup on your raw data sheet, and make a note of the reference height (considered to be zero). Make sure to label all parameters in your sketch.
5. To locate the point of impact, launch the projectile for a few preliminary trials.
6. Tape a piece of paper on the impact area and place a carbon sheet on top of it. **Do not tape down the carbon sheet!**
7. Launch the projectile for three separate trials. Before each launch, check that the launch angle is still correct as it might change after you set the launch range. Be sure that the carbon sheet remains on the paper covering the impact area throughout the trials. The projectiles should leave a carbon imprint on the paper at their impact points.
8. Remove the carbon sheet but **do not remove** the taped down paper. Determine the average impact point on the paper and mark it. To record the range, $x - x_0$, measure the horizontal distance between the launcher mouth and the average impact point.
9. Repeat the experiment with a different set of parameters provided by your TA.

3.2.3 Angle of Maximum Range

1. Relocate and adjust the launcher so that the launch and impact heights are the same, that is $y = y_0$. Use the provided boxes to set the heights equal. You will need to use a notebook or pieces of paper to adjust for any differences that can occur for each angle setting.
2. Use the **medium** setting for the launcher. Set the launch angle to 40°.
3. Launch the projectile three times and mark the average impact point. You may use the same technique of marking this average impact point as done previously.
4. Measure and record the range, $x - x_0$, which is the horizontal distance between the launcher mouth and the average impact point.
5. Repeat, at the launcher's medium setting, for a launch angle of 45° and again for 50°. Make sure to measure y and y_0 to ensure the heights are still equal.

3.3 Analysis

i. Calculate the average muzzle speed, v_0 for both range settings. Show your work.

ii. For the two launch experiments, find the theoretical ranges ($x - x_0$) using the experiments' parameters and muzzle velocities. Do this by solving equation (3.8) for the time t, and then plug it into equation (3.7). **You will need to use the quadratic formula.** Show your work.

iii. Compare (by relative error) your theoretical ranges to your measured ranges. Comment on any differences between the theoretical and the measured. Were the effects of air friction noticeable? Explain.

iv. From equations (3.7) and (3.8) show, in general, that the range is given by $R = (V^2/g)\sin(2\theta)$, where V is the initial speed of the projectile, g is the acceleration of gravity, and θ is the launching angle.
Hint: use the identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

v. Make a plot of the range R as a function of the launching angle θ.

vi. Does your data support the prediction that the maximum range is attained at 45°?

Lab 4

Newton's Laws

4.1 Background

In this lab, we use a computerized system for data acquisition to measure velocity, acceleration, force, and the coefficients of friction. We make these measurements for several applications: motion through a pulley system called an Atwood machine, along an air track, across a table surface, and down an inclined plane.

This lab spans two weeks. The first week involves becoming familiar with the software and sensors of the computerized system, and conducting experiments using the air track. The second week includes collecting data for different experiments that involve several different sensors and setups.

This lab mainly demonstrates Newton's second and third laws of motion. Newton's second law states that the net force, $\vec{F}_{\text{net}}$, acting on a body is directly proportional to the object's mass, m, and its acceleration, $\vec{a}$. Thus the equation is given by $\vec{F}_{\text{net}} = m\vec{a}$. Newton's third law states that any action will have an equal and opposite reaction. In the following subsections, the theory associated with each experiment is described.

4.1.1 Acceleration along an Air Track

In this experiment, we find both the acceleration of the car and the total frictional force acting against it as the car is pulled by a falling mass. The basic components of the experiment are illustrated in Figure (4.1).

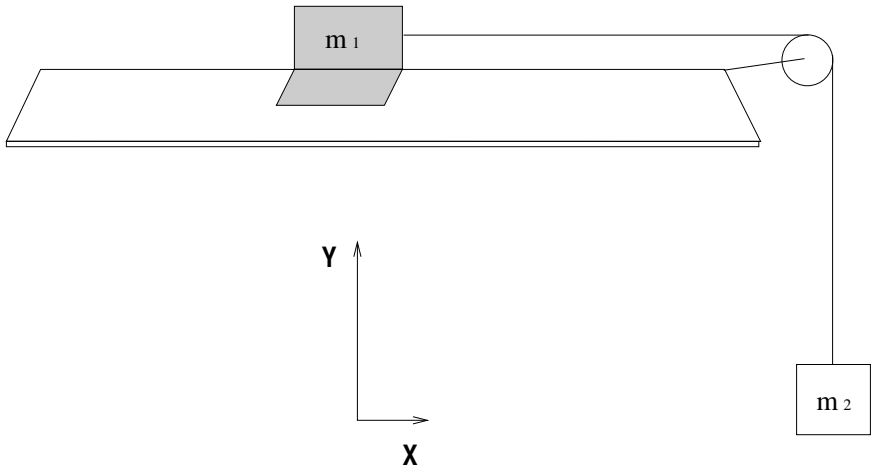

Figure 4.1: Diagram demonstrating the basic components of the experiment. Note that gravity, $\vec{g}$, is pointing down vertically.

Applying Newton's laws, we have

$$\begin{array}{ll} F_{1,\text{net}} = T - f & F_{2,\text{net}} = m_2 g - T \\ m_1 a = T - f & -m_2 a = T - m_2 g \end{array}$$

where f is the net frictional force opposing the car's motion.

By Newton's third law, the tension felt by m_1 must be equal (and opposite) to the tension felt by m_2. Therefore, we find the acceleration by solving both equations for the tension and then setting them equal to each other:

$$m_1 a + f = m_2 g - m_2 a$$

$$(m_1 + m_2)a = m_2 g - f$$

$$a = \frac{m_2 g}{m_1 + m_2} - \frac{f}{m_1 + m_2}$$

$$a_{\text{measured}} = a_{\text{ideal}} - \frac{f}{m_1 + m_2} \tag{4.1}$$

where a_{ideal} is the acceleration of the car in the absence of friction and given by

$$a_{\text{ideal}} = \frac{m_2 g}{m_1 + m_2} \tag{4.2}$$

In the experiment, we will find the total acceleration a_{measured} and then calculate the frictional force f in the **Analysis Section** according to

$$f = (m_1 + m_2)(a_{\text{ideal}} - a_{\text{measured}}) \tag{4.3}$$

which is obtained from equation (4.1) by solving it for f.

4.1.2 String Tension along an Air Track

This experiment is essentially the same as the previous except that now the goal is to find the tension in the string rather than the acceleration.

Referring to the diagrams and application of Newton's laws in the previous experiment, we find the tension by solving both equations for a and then setting them equal to each other as follows:

$$\frac{T - f}{m_1} = \frac{m_2 g - T}{m_2}$$

$$\left(\frac{1}{m_1} + \frac{1}{m_2}\right) T = g + \frac{f}{m_1}$$

$$T = \left(\frac{m_2}{m_1 + m_2}\right)(m_1 g + f)$$

The only quantity unknown in this equation is the frictional force f. This value must be calculated from the measured acceleration of the car according to equation (4.3).

Substituting the expression for f and simplifying, we have

$$T = \left(\frac{m_1 m_2}{m_1 + m_2}\right) g + m_2(a_{\text{ideal}} - a_{\text{measured}})$$

$$T = \left(\frac{m_1 m_2}{m_1 + m_2}\right) g + m_2 \left[\left(\frac{m_2 g}{m_1 + m_2}\right) - a_{\text{measured}}\right]$$

$$T = m_2(g - a_{\text{measured}}) \tag{4.4}$$

This final expression shows that the "missing" acceleration (that is, the observed difference from the gravitational acceleration) goes entirely into the tension of the string. Alternatively, stopping the car (i.e acceleration is zero) gives $T = m_2 g$. When the car is released, the tension decreases. Therefore, the physical mechanism of the tension while the car is moving is due partly to frictional force and partly to the inertial mass of the car.

In the experiment, we measure the tension in the string directly and then compare it in the **Analysis Section** to that predicted by equation (4.4).

4.1.3 Acceleration through an Atwood Machine

In this experiment, we find the acceleration of a mass as it "falls". The acceleration is not the same as the gravitational acceleration g because the mass is attached to another, lighter mass through a string that is threaded across the pulleys as shown in Figure (4.2).

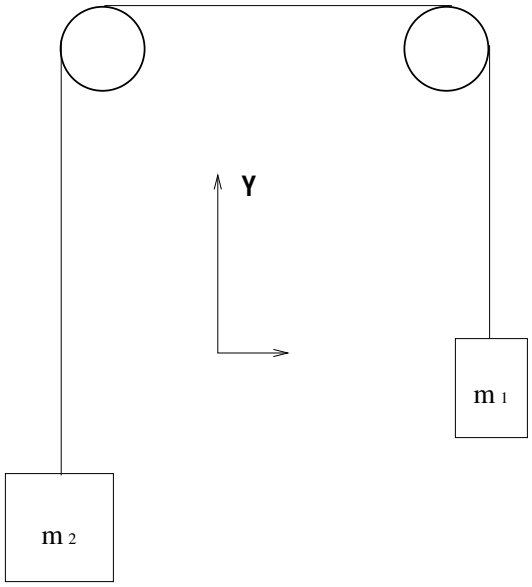

Figure 4.2: Schematic showing the basic outline of an Atwood Machine.

Applying Newton's laws, we have

$$\begin{aligned} F_{1,\text{net}} &= T - m_1 g & \qquad F_{2,\text{net}} &= T - m_2 g \\ m_1 a &= T - m_1 g & \qquad -m_2 a &= T - m_2 g \end{aligned}$$

where a is the acceleration and T is the tension in the string.

To find the acceleration, we repeat the same process from before of solving both equations for tension and then setting them equal to each other to get

$$m_1 a + m_1 g = -m_2 a + m_2 g$$

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g \tag{4.5}$$

Notice that we can make the acceleration a much smaller than g by significantly decreasing the difference between m_2 and m_1. But we can also change a while keeping the difference the same; we do this by adding masses to both sides such that the sum of them in the denominator increases while the difference in the numerator remains constant.

In the experiment, we vary the masses on the Atwood machine and measure the different accelerations. In the **Analysis Section**, we will use relative errors to compare the measured values to those predicted by equation (4.5).

4.1.4 Static Friction on a Horizontal Surface

We now introduce the concepts of kinetic and static friction. As their names imply, kinetic friction applies to moving objects, while static applies to those at rest. For instance, when you try to move an object, but a small amount of applied force results in no motion, the force of friction is greater than the applied force, and it is called static friction. However, if we keep increasing the value of the applied force, it eventually will match the value of the static friction, and the object begins to move. The point at which the applied force overcomes the static friction is called the "break away" point. It is observed that at this point, the force required to balance the friction and keep the object moving at constant speed is smaller that the force needed to initiate the motion. This reduced friction observed in moving objects is called kinetic friction. Figure (4.3) shows the magnitude of the frictional force as a mass is pulled across a surface. In this experiment, we explore static and kinetic friction by steadily pulling a mass on a horizontal surface until it begins to move.

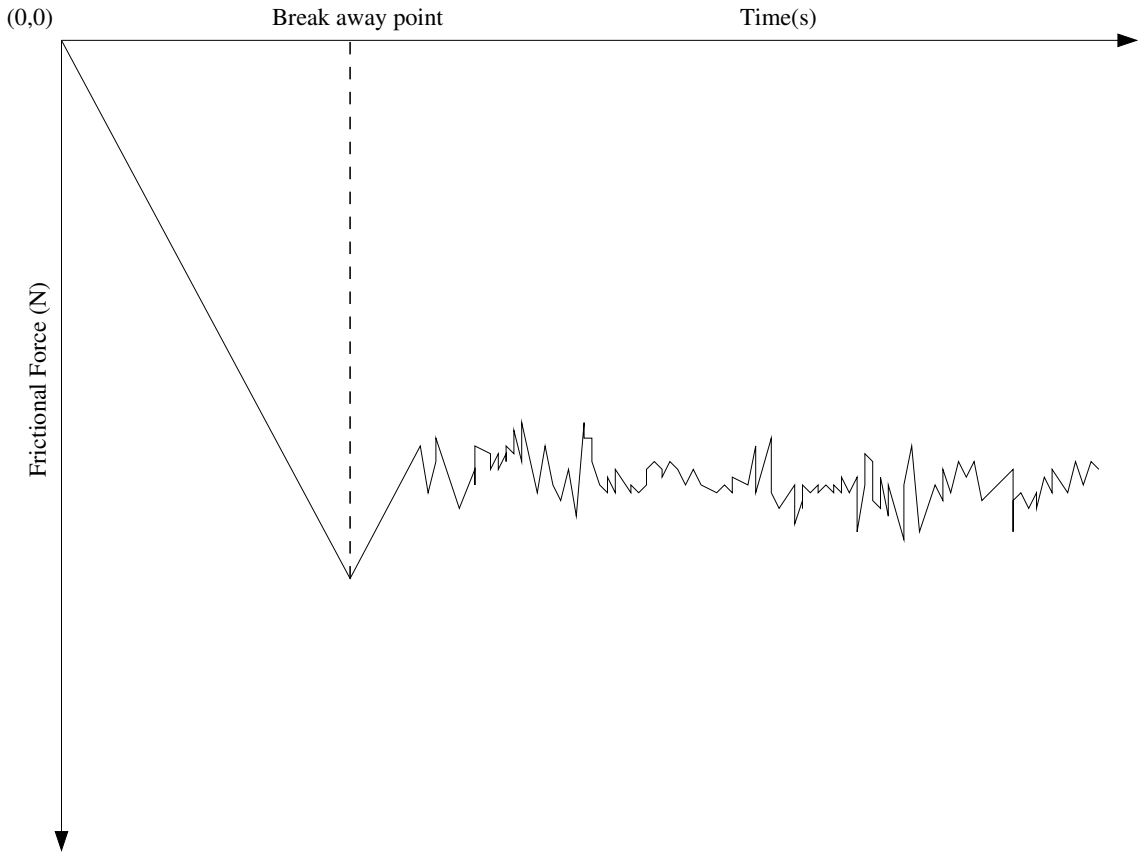

Figure 4.3: Schematic showing the break away point. Prior to the break away point, we are in the static region, while afterwards we are in the kinetic region.

We obtain our applied force directly from the plot, which allows us to calculate the maximum coefficient of static friction at the break away point. We use the following equations:

$$f_s = |\vec{F}_{\text{applied}}|$$

$$\mu_s|\vec{N}| = |\vec{F}_{\text{applied}}|$$

$$\mu_s mg = |\vec{F}_{\text{applied}}|$$

$$\mu_s = \frac{|\vec{F}_{\text{applied}}|}{mg} \tag{4.6}$$

where f_s is the static frictional force, N is the normal force (directly perpendicular to the surface), and μ_s is the coefficient of maximum static friction.

4.1.5 Kinetic Friction on an Inclined Surface

In this experiment, we find the coefficient of kinetic friction by measuring the acceleration of a mass as it slides down an inclined plane. Like the other experiments above, the acceleration measured is not the same as the gravitational acceleration g. In this case, however, the diminishing of the acceleration is partly due to the incline of the plane and partly due to the kinetic frictional force (see Figure (4.4) for an illustration).

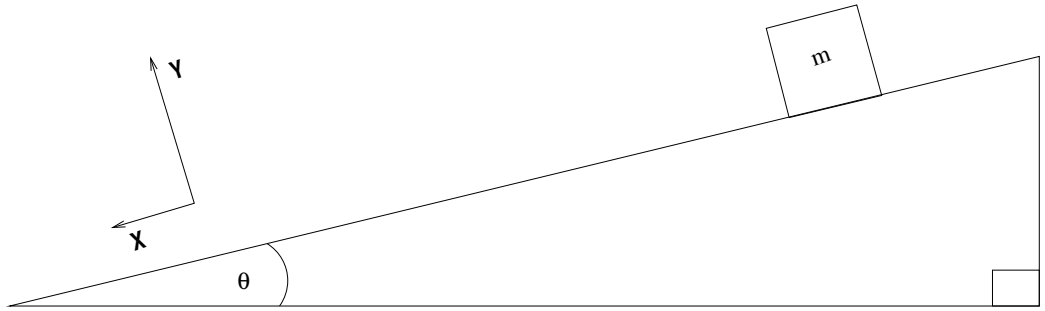

Figure 4.4: Schematic showing a mass sliding down an inclined plane.

To find the coefficient of kinetic friction, μ_k, we apply Newton's laws to the above arrangement and separate into Cartesian components.

$$\begin{aligned} F_{\text{net,x}} &= mg\sin(\theta) - f_k & \qquad F_{\text{net,y}} &= -mg\cos(\theta) + N \\ ma &= mg\sin(\theta) - f_k & \qquad 0 &= -mg\cos(\theta) + N \end{aligned}$$

The friction coefficient is encapsulated in the frictional force f_k that appears in the x-component of the net force equation. We use the y-component to find the expression for the normal force in terms of known parameters and substitute this into the definition of f_k:

$$f_k = \mu_k N$$

$$f_k = \mu_k mg\cos(\theta)$$

We now substitute this into the x-component of the net force equation and solve for the friction coefficient. After simplifying, we have

$$ma = mg\sin(\theta) - \mu_k mg\cos(\theta)$$

$$\mu_k = \tan(\theta) - \left|\frac{a}{g}\right|\sec(\theta) \tag{4.7}$$

Notice the absolute value bars in this equation. This is because the acceleration we find is a negative value. The kinetic frictional force is **always less than** the static one, which implies that $\mu_k < \mu_s$.

In the experiment, we measure the acceleration of the mass and then use equation (4.7) to calculate the coefficient of kinetic friction.

4.2 Procedure

To familiarize yourself with the motion sensor and graphing software, first conduct the exercise, **Velocity, and Acceleration of Students**. Then the two experiments involving the air track will be performed during this first week, with the corresponding analysis being completed during the lab session. The other experiments and their corresponding analyses will be done and completed the following week during the lab session. **Check the "Useful Guidelines" Handout for help during the laboratory or inquire with your TA when needed.**

4.2.1 Velocity and Acceleration of Students

This exercise uses the motion sensor to measure velocity. It measures the distance to an object from the delay times of reflected ultrasonic "chirps" that it emits. It sends this information to the computer where the software calculates the velocity of the object and plots it against time. The acceleration is calculated from the slope of this plot.

1. Set the sensor to BROAD BEAM.
2. Walk in the path of the beam; try changing your speed occasionally.
3. Examine the distance and velocity graphs displayed on the computer. Try to familiarize yourself with the software and learn how to edit the data points.

4.2.2 Acceleration along an Air Track

This experiment uses the motion sensor to measure the acceleration of a car along an air track as it is pulled by a falling mass. Refer to the diagram in the corresponding subsection of the **Background Section** for illustration of the basic setup and parameters.

1. Place the four "silver" masses (each of mass 50 g) on the sides of the car.
2. Weigh the mass m_1 of the entire car and separately weigh the mass m_2. The mass m_1 of the entire car includes the car itself, the four masses added to it in step 1, and the sail attached to its top; m_1 should be approximately 0.40 kg. The mass m_2 is the hanging weight that falls to provide the acceleration for the car; it should be approximately 0.02 kg. Make sure to record *your* values for m_1 and m_2!
3. Place the car on the air track, attach the string, and thread the other end of the string over the pulley.
4. Adjust the sail so that it is perpendicular to the direction of the car's motion.
5. Align the motion sensor with the track and set it to NARROW BEAM. Be sure that the sensor is facing the car head on and that its beam is perpendicular to the sail and reflects off it. Check the setup by monitoring the software's output while running the car along the track.
6. Attach the mass m_2 to the free end of the string so that it hangs down. Hold the car in place to keep the system still.
7. Turn on the air track pump and give it a few seconds to allow for even distribution of air within the track.
8. Set the software to acquire the data using the start button. Release the car.
9. In the software, select the plot of velocity with respect to time.
10. Find the acceleration from the velocity plot; record this value. For this, use the same method as in the previous experiment: isolate a straight portion of the graph and apply the Linear Fit function to get the equation of the best-fit line, where the slope is the acceleration. Print out one copy of the plot and label it accordingly.

    In the **Analysis Section**, your measured value is compared to that predicted by equation (4.2), and then both values are used to calculate the total frictional force according to equation (4.3).

4.2.3 String Tension along an Air Track

This experiment is the same as the previous except that we now also measure the tension in the string with the force sensor. The setup is nearly the same, but the masses are different as seen in the following steps. Refer to the diagram in the **Background Section**.

1. Fit the force sensor onto the top of the car and place the sail on top of the sensor. Make sure that the car also has the four 50 g (silver) masses attached to its sides.
2. Weigh the mass m_1 of the entire car and separately weigh the mass m_2. The mass m_1 of the car should be approximately 0.48 kg. The mass m_2 is the hanging weight that produces the tension that pulls the car; it should be approximately 0.10 kg. Make sure to record *your* values for m_1 and m_2!
3. Place the car on the air track and tie the string to the hook on the force sensor; thread the other end of the string around the pulley.
4. Align the motion sensor and sail. The sail should be perpendicular to the car's direction of motion, and the motion sensor's beam should shine directly on the sail. Be sure that the motion sensor is set to NARROW BEAM. Note that this might require that the motion sensor itself is raised slightly up to reflect off the sail once more.
5. Attach the mass m_2 to the free end of the string so that it hangs freely. Be sure to hold the car to keep the system still.
6. Turn on the air track pump and give it a few seconds to allow even distribution of air within the track.
7. Set the software to plot velocity against time for the motion sensor **and** to plot force against time for the force sensor; begin recording data.
8. While holding the car still, lift up the mass m_2 to slacken the string. Push TARE on the force sensor to set the force sensor to zero for the system.
9. Gently lower the mass m_2 while still holding the car motionless. Make sure that the force plot shows the tension equal to the value of $m_2 g$ (≈ 0.98 N).
10. Release the car.
11. Find and record the value of the acceleration by the same method as in the previous two experiments.
12. Record the value of the tension, which is measured directly from the force plot. Be sure to measure the plotted tension for the duration of motion not for any times before or after that.
13. Print out one copy of the velocity plot and the force plot and label them accordingly.

    In the **Analysis Section**, your measured value for the tension is compared to that predicted by equation (4.4). The accelerations are also compared, and the frictional force is calculated as well.

4.2.4 Acceleration through an Atwood Machine

This experiment uses a smart pulley to measure the acceleration of a falling mass. The smart pulley is a photogate sensor that detects the interruptions of its infrared beam that it projects through the spokes of the pulley. The information is sent to the computer where the software plots it as velocity against time.

Perform three trials of this experiment with the following parameters:

	m_1 (kg)	m_2 (kg)
Trial 1	0.250	0.260
Trial 2	0.250	0.270
Trial 3	0.250	0.280

1. Place 0.260 kg (m_2) on the end of the string that is threaded around the smart pulley and 0.250 kg (m_1) on the other end. Hold the string while attaching the masses to keep the system still.
2. Set the software to acquire data.
3. Place the cushion below m_2. Release the string when m_2 is close to the smart pulley sensor.
4. Once the data is acquired by the software, select the plot of velocity with respect to time.
5. Isolate a clean, straight portion of the velocity plot. This requires using the editing features of the software.
6. Apply the Linear Fit function to the isolated portion. The computer calculates the equation of a best-fit line for the data.
7. Record the slope of the best-fit line. This slope is the downward acceleration of the mass m_2. In the **Analysis Section**, this value is compared to that predicted by equation (4.5). This method for finding the acceleration is much better than having the software automatically calculate it from the distance data, which tends to produce a noisy trace. Print one copy of the plot and label the printout accordingly (e.g., Atwood Machine Trial 1).
8. Repeat for the other two trials, including a printout for each.

4.2.5 Static Friction on a Horizontal Surface

This experiment uses the force sensor to measure the maximum amount of force applied to a mass before "break away" on a horizontal surface. From this maximum force, the maximum coefficient of static friction can be calculated.

1. Record the material of your chosen horizontal surface. Do not worry about the specific composition; just write the common name for it, for example "cardboard".
2. Fit the force sensor onto the top of the 2.0 kg mass. The mass has a slot that holds the post needed to attach the force sensor. Be sure to place it such that the sensor remains attached to the mass while being pulled.
3. Attach the string to the force sensor and push TARE to set the force sensor to zero for the initial conditions of the system.
4. Set the software to plot force against time and begin recording.
5. Hold the string lightly and then gradually pull to increase tension until the mass breaks away.
6. Record the maximum force displayed on the plot. In the Analysis, this value is used to calculate the maximum coefficient of static friction by equation (4.6). Print out one copy of the plot and label it accordingly.
7. Repeat the experiment with a different type of material. Make sure to record the new type of material used.

4.2.6 Kinetic Friction on an Inclined Surface

This experiment uses the motion sensor to measure the acceleration of a wooden block as it slides down an inclined plane. From the value of the acceleration and inclination angle, the coefficient of kinetic friction can be calculated.

1. Record the inclination angle, θ, of your ramp.
2. Align the motion sensor at the top of the inclined plane and set it to NARROW BEAM. Be sure that the beam shines parallel to both the planar surface and the path of the sliding block.
3. Place the block on the inclined plane near the top, in front of the motion sensor. Hold it in place while you prepare the data acquisition software. You do not need to know the mass of the block for this experiment.
4. Release the block. Be sure that the software is acquiring your data.
5. Select the plot of velocity with respect to time in the software.

6. Find and record the value of the acceleration. Use the same method outlined in the air track experiments above: isolate a straight portion of the graph and apply the Linear Fit function to get the equation of the best-fit line, where the slope is the acceleration. Print one copy of the plot and label it accordingly.

   In the **Analysis Section**, your measured value of the acceleration and the angle of inclination are used in equation (4.7) to calculate the coefficient of kinetic friction.

4.3 Analysis

i. For the experiment, Acceleration through an Atwood Machine, use equation (4.5) to calculate the theoretical value of acceleration for each trial. Compare, using relative error, your theoretical values to your measured values.

   The magnitude of the relative error should decrease as the difference between m_1 and m_2 increases. Does it? Explain.

ii. For the experiment, Acceleration along an Air Track, use equation (4.2) to calculate the theoretical acceleration for the measured masses. Compare these values to each other using relative error.

   The percentage of error is the fraction of the force that was counteracted by frictional effects (friction of the track and pulley and air drag on the sail). Use equation (4.3) to calculate the total frictional force.

iii. For the experiment, String Tension along an Air Track, repeat the analysis of part ii above. Then use equation (4.4) to calculate the theoretical tension in the string. Compare this value to the one measured directly by the force sensor; use the relative percentage error.

iv. For the experiment, Static Friction on a Horizontal Surface, use your measured value of the maximum force in equation (4.6) to calculate the coefficient of static friction.

v. For the experiment, Kinetic Friction on an Inclined Surface, use your measured value of the acceleration and the corrected inclination angle in equation (4.7) to calculate the coefficient of kinetic friction. You may have measured a negative value for the acceleration; use the absolute value in your calculation.

Lab 5

Conservation of Energy

5.1 Background

Conservation of energy is used in this lab to make a prediction of the exit speed of a car "sliding down" a fancy ramp (see Figure 5.1). The actual exit speed is then used to calculate the landing spot for the car. We assume that as the car moves down the ramp, some part of the potential energy is dissipated by friction, but the rest is converted only into translational kinetic energy. In particular, no energy is expended in the rotation of the car's wheels. We further assume that air drag can be neglected, both over and off the ramp.

5.2 Procedure

1. Figure 5.1 shows the setup for this lab. Use this figure to do the energy analysis for predicting the speed of the car at the end of the ramp; assume that the initial speed is zero and include a term for the frictional dissipation.

2. Carry out the analysis to predict the horizontal distance traveled by the car between the end of the ramp and the landing spot. Assume that the initial velocity is known.

3. Wait for your turn to use the ramp and measure the height of the point of the ramp from where the car is launched; the height at the exit point; the car's length; the ramp's exit angle.

4. Make sure that the photogate is set to "gate mode." In this mode, the photogate reports the time that its beam is blocked, which occurs when the car is passing through it.

5. **Before you launch the car be ready to catch the car with the cushioned bucket.**
   Carefully launch the car and measure the time it takes for the car to pass through the photogate.

6. Go back to the table and use your data to produce a numerical value for the location of the landing spot.

7. Wait for your turn to test the prediction. Measure the actual distance to the landing point.

5.3 Analysis

i. Show the analysis that leads to a formula for the prediction of the exit speed. **Wait** until you have the formula to plug any numbers.

ii. Compare your prediction with the measured value.

iii. Give an estimate for the energy lost to friction over the ramp.

iv. Show the projectile motion analysis that leads to a formula for the prediction of distance X in Figure 5.1; **Wait** until you have the formula to plug numbers. Use the value of the measured exit speed to calculate X. Enclose your numerical value with appropriate units.

v. Compare your prediction of distance to the landing spot to the measured value.

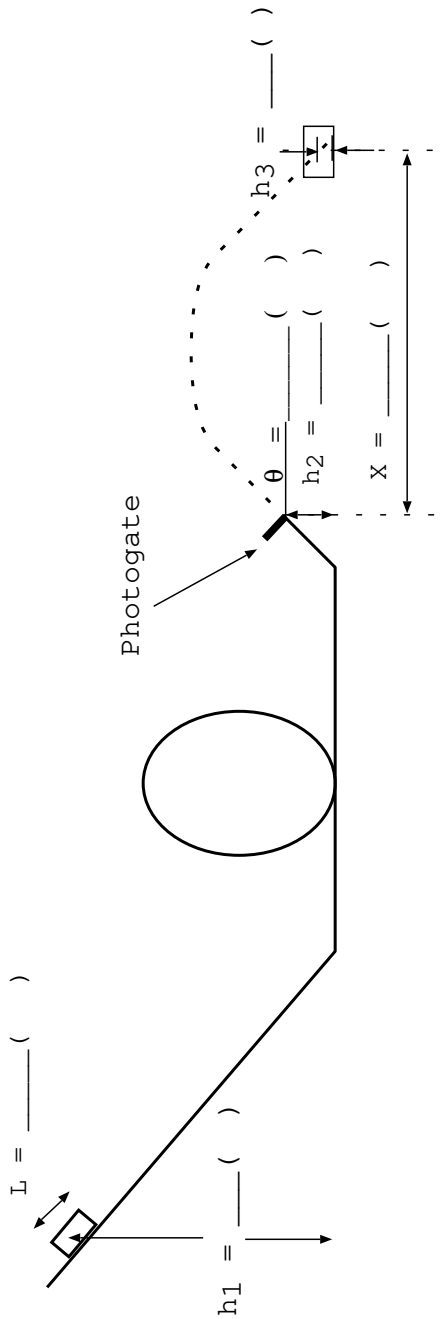

Figure 5.1: This figure shows the setup and the measurements required for the analysis.

Lab 6

Binary System

6.1 Background

In this lab, we study the conservation of energy and momentum in a binary system, which consists of two similar objects of equal mass moving on an air table, and coupled by a spring. The frictional losses are small, and the total energy of the system is well approximated by the sum of the kinetic and potential energies. In our system, the mass of the spring is neglected in comparison with the mass of the objects. Therefore, the kinetic energy is only associated with the motion of the masses. Since the objects move on a horizontal plane, their gravitational potential energy remains unchanged. On the other hand, as the spring connecting the masses stretches and contracts, it will store and release energy. This elastic potential energy is represented by

$$\frac{1}{2}\kappa(r - r_0)^2 \tag{6.1}$$

where r is the length of the spring at any time, and r_0 represents the length of the relaxed spring; κ is the spring constant, which indicates the spring stiffness. Since this is an ideal spring, its force is conservative and you can show that equation (6.1) is consistent with Hook's law

$$F = -\kappa(r - r_0). \tag{6.2}$$

The total energy of two interacting objects is given by

$$E_{\text{total}} = K_1 + K_2 + V_g(h) + V_s(r), \tag{6.3}$$

where $K_1 = \frac{1}{2}m_1v_1^2$, and $K_2 = \frac{1}{2}m_2v_2^2$ are the kinetic energies of the two objects; $V_g(h) = m_1gH + m_2gH$ is the gravitational potential energy; $V_s(r)$ represents the elastic potential energy of the interaction, and it is given by equation (6.1).

As long as the frictional losses are small, the total energy of the system should be constant. Furthermore, in our experiment the gravitational potential energy only adds an constant value to the total energy and it can be ignored. Therefore, we will write the total energy of the system as

$$E_{\text{total}} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}\kappa(r - r_0)^2 \tag{6.4}$$

We will use this equation to test the conservation of energy in our system.

Conservation of momentum states that, if there are no outside influences on the system, the momentum of its center of mass will not change. Therefore,

$$\vec{p}_{\text{total}} = m_{\text{total}}\vec{v}_{cm} = \text{constant} \tag{6.5}$$

where $\vec{v}_{cm}$ is the velocity of the center of mass. This equation shows that the center of mass moves at a constant speed and in a constant direction.

6.2 Procedure

The following steps provide direction to set up the equipment, perform the experiment, and prepare the data for analysis. However, it is necessary to perform a preliminary experiment to determine the spring constant, κ.

6.2.1 Spring Constant

1. Hook the spring to a stand and attach a weight hanger to the bottom. Note that the weight hanger has a mass of 50 g.

2. Measure and record the relaxed length, l_0, of the hanging spring.

3. Add a small mass m to the hanger and measure the resulting length l of the spring; record both the mass and the length in your raw data sheet.

4. Repeat step 3 for **four** additional small increments of mass. Fill in a table that shows all of your values.

   In the **Analysis Section**, this data is used to calculate the spring constant, which is needed for calculating the potential energy of the system.

6.2.2 Equipment

1. Note that the mass of each puck is 556 g.

2. Place a large sheet of spark paper on the air table. It should cover the surface completely.

3. Place the pucks on the air table and connect them with *your* spring.

4. Turn on the air pump for the table.

5. Note that the frequency for the spark timer is 20 Hz.

6.2.3 Experiment

1. Make sure that the air table is leveled.

2. Position the pucks near an edge on the table; this is the starting position of the binary system.

3. Practice pushing the pucks such that they orbit each other as they drift across the table. Give the orbit sufficient eccentricity; that is, make it somewhat elliptical so that the separation between the pucks varies measurably as they drift. If needed, ask your TA for guidance.

4. Set the pucks in motion as practiced and quickly activate the spark timer using the foot pedal. Be sure to turn off the spark timer as soon as one of the pucks collides with the edge of the air table. Doing so allows you to identify more easily the pairing of dots.

5. At the end of the run, remove the spark sheet from the air table. **Make sure to mark the starting point on your spark sheet.** Confirm your result with your TA before moving on.

6. To obtain the relaxed distance, r_0, between the pucks, first reposition the pucks to one of the corners of the spark paper that does not contain your binary system track. Next, make sure that the spring is not significantly stretched or compressed and that the pucks are stationary. Finally, activate **briefly** the spark timer.

7. Turn off the spark timer and power down the air pump if no other group is using it.

6.2.4 Measurements

1. Measure and record the distance between the two dots created away from the binary system paths. This distance is the equilibrium distance, r_0 or, which is needed for calculating the potential energy of the system.

2. Label the paths of the binary system as path a and path b. Index every third dot on each path. That is, start from the first dot on the paths, and label it as 0. Then skip two dots, and label the next dot as 3, skip another two dots, and label that dot as 6, and so on. Do the same for the other path. These labels indicate corresponding times of travel for each of the pucks. Check with your TA if you are unclear about how to start the sequencing and labeling.
   You may want to use two different colors for the indices to distinguish between the two pucks, making the following measurements easier.

3. Measure and record the distances between successive indexed dots of path a. Record these distances as Δa in a table with their corresponding time index. These values are used in the **Analysis Section** to calculate the puck velocity.

4. Repeat for the corresponding dots of the path b. Record these distances as Δb in a table with their corresponding time index.

5. Now measure and record the separation distance r between dot pairs with the same index, e.g., the distance between dots labeled 0 on both paths. These values of r are used in the **Analysis Section** to calculate the potential energy of the puck system according to equation (6.1).

6. Construct a sequence of the center of mass points. Because the pucks have the same mass, the center of mass is halfway between the dots of a pair. Connect dot pairs with a straight line, e.g., dots labeled 0 on both paths. For each line, mark the center of the line.

7. Measure the distance between each successive center of mass point; record this distance as x in your table with its corresponding time index. In the **Analysis Setion**, these distances and time intervals are used to calculate the velocity of the center of mass, $\vec{v}_{\rm cm}$.

6.3 Analysis

i. Find the spring constant in Hook's law, $|F| = \kappa x$. This law is a linear relationship between the deformation x (distance from equilibrium) and the applied force, F. Plotting F on the vertical axis and x on the horizontal axis results in a straight line, whose slope represents the spring constant, κ. If our spring follows Hook's law, our data should resemble a straight line, although not exactly –perhaps due to measurement errors.
   Plot your data with the weight on the Y axis, and the distance from equilibrium on the X axis. Use a ruler to draw the best-fit line through your data. Remember that the best-fit line should be close to all of your data, except for the obvious outliers. Determine the slope of the best-fit line.

ii. Calculate the kinetic energy for each puck and record them in your table. Recall that the spark frequency of 20 Hz implies a time of 0.05 seconds. Since you measured every third dot, the change in time, Δt, is 0.15 seconds. Hence, to determine v_a and v_b use $\Delta a/\Delta t$ and $\Delta b/\Delta t$, respectively, where Δt = 0.15 seconds. Also, record the total kinetic energy.

iii. Calculate the elastic potential energy term in equation (6.1) for each measured value of the puck separation distance r and record them in your table.

iv. Find the total energy according to equation (6.4) for each indexed position and record them in your table.

v. Plot the kinetic energy of each puck and their sums against time Make sure to connect the dots with a line and label each line clearly.
   Explain the dynamics of these curves.

vi. Make a second plot of the total kinetic energy, elastic potential energy, and total energy against time.
   Explain the dynamics of these curves and relate them to the principle of the conservation of energy.

vii. From these plots, do you think that we have properly accounted for the potential energy of the spring? What might cause the total energy to be misestimated? Explain!

viii. Does the second plot show the total energy as constant? If not, determine the percentage of the initial energy that was lost and explain what happened to it.

ix. Calculate the velocity of the center of mass using your measured distance, x, and time intervals (i.e., Δt = 0.15 sec) between the center of mass points. Are your measurements consistent with equation (6.5) or does the velocity vector change along the trajectory? If so, by how much? Explain any deviations.

Note: follow your instructor's directions concerning the submission of your spark sheet with your lab report.

Lab 7

Collisions in 1-D and 2-D

7.1 Background

In this lab, we investigate momentum and energy conservation in collisions. We use two cars on an air track to demonstrate one-dimensional elastic and inelastic collisions, and we use two pucks of equal mass on an air table to demonstrate two-dimensional elastic collisions.

Each of the following subsections examines specific cases of the general principles of momentum and energy conservation. For the momentum of two masses, we have

$$\vec{p}_i = \vec{p}_f \tag{7.1}$$

or

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}. \tag{7.2}$$

For the kinetic energy of two masses, we have

$$K_i = K_f + Q \tag{7.3}$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 + Q, \tag{7.4}$$

where Q is the energy lost to frictional and other effects (and usually goes into heat).

7.1.1 One-Dimensional, Perfectly Elastic Collision

In this case, Q in equation (7.4) is zero, and we can use the two equations to find the velocities of the two masses after collision (two equations and two unknowns). To simplify further, we consider the case of equal masses (i.e. $m_1 = m_2$) with one initially stationary. Then equations (7.2) and (7.4) become, respectively,

$$v_{1i} = v_{1f} + v_{2f} \tag{7.5}$$

and

$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2. \tag{7.6}$$

Substituting v_{1i} from the first equation into the second and simplifying, we get

$$2v_{1f}v_{2f} = 0. \tag{7.7}$$

Because v_{2f} cannot be zero (assuming m_2 is in the way of m_1), we conclude that v_{1f} is zero and that all the energy went into the second mass. That is, after the collision, the first mass stops, and the second mass moves with the same initial velocity of the first mass.

7.1.2 One-Dimensional, Partially Elastic Collision

In this case, Q is not zero. If Q is known and the momentum is conserved, we can find v_{1f} and v_{2f} with equations (7.2) and (7.4) using the known values of v_{1i}, v_{2i}, m_1, and m_2. Alternatively, if we do not know the value of Q, we can find it along with one of the final velocities as long as we know all the other parameters.

However, momentum is not fully conserved in our lab experiments because of frictional losses. However, we could assume that momentum is closely conserved if the collision time is very small. In this experiment, we test the *accuracy* of our assumption about momentum conservation and find the energy loss, Q.

7.1.3 One-Dimensional, Totally Inelastic Collision

In this case, the two masses stick together upon colliding. Therefore, assuming that momentum is conserved, we find the final velocity from equation (7.2) alone. If the second mass is initially stationary, we have from equation (7.2)

$$m_1 v_{1i} = (m_1 + m_2) v_f \tag{7.8}$$

$$v_f = \left(\frac{m_1}{m_1 + m_2} \right) v_{1i} \tag{7.9}$$

7.1.4 Two-Dimensional Elastic Collision

This case is just like the single-dimensional one except that now both masses move in two dimensions. Assuming conservation of momentum, we use the vector form of equation (7.2) and separate it into its Cartesian components to get

$$m_1 v_{1,ix} + m_2 v_{2,ix} = m_1 v_{1,fx} + m_2 v_{2,fx}, \tag{7.10}$$

where $m_1 = m_2$. Because the two masses are equal in the experiment, they cancel in the equation. Hence,

$$v_{1,iy} + v_{2,iy} = v_{1,fy} + v_{2,fy} \tag{7.11}$$

Due to the elastic collision, the initial velocities of each puck should be equivalent to the final velocities of each puck after the collision.

7.2 Procedure

The first three experiments that follow use the same setup of the air track and two photogate timers. Make sure that the timers are well separated from each other and hang over the track so that the sails (flags) on the cars can pass through completely.

7.2.1 One-Dimensional, Perfectly Elastic Collision

1. Make sure that the air tracks are leveled.
2. Place two cars of equal mass on the air track and insert a sail on the top of each. **Note:** All the accessories (such as sails and bumpers) are about the same mass, so placing equal numbers of accessories on the cars should keep the cars' masses equal.
3. Measure and record the length of the sail. This length is needed to determine the velocities of the cars as they pass through the photogates, which measure the period of beam interruption.
4. Insert the rubber band bumper into one of the two facing sides of the cars. This elastic bumper, along with the nearly frictionless air track surface, makes the collisions almost perfectly elastic.
5. Position the second car between the two photogates such that the first car can completely pass through the first photogate before the collision. Note that the second car has to be motionless in its position!

6. Position the first car at the beginning of the track. Make sure that its sail has a little distance from the photogate. See Figure (7.1) for the setup. Notice that $m_1 = m_2$ in this case.

7. Set the photogate timers to GATE mode.

8. Gently and briefly push the first car toward the second. Be sure to stop pushing before the sail reaches the photogate.

9. Record the times in the appropriate part in the raw data sheet.

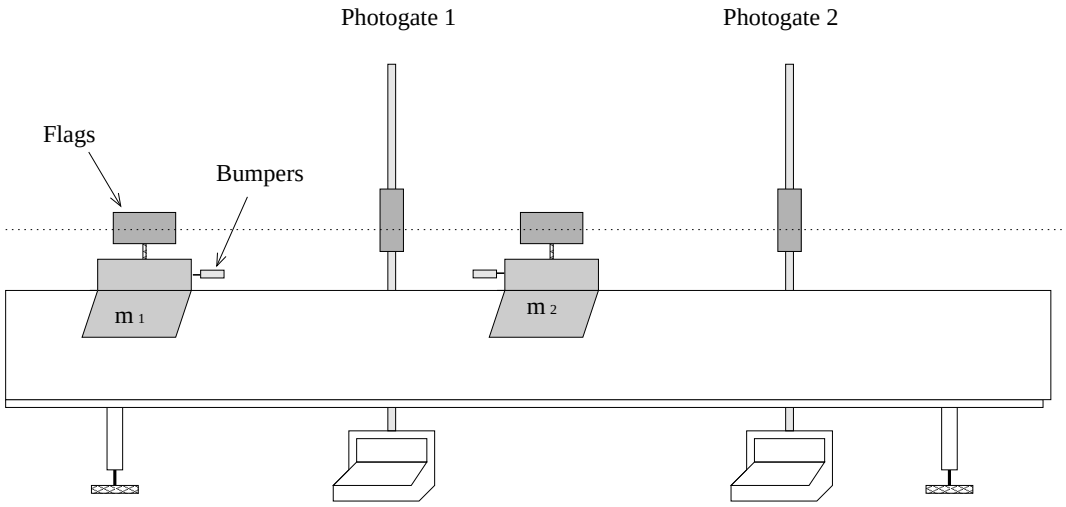

Figure 7.1: Schematic showing the setup for the 1-D perfectly elastic collision experiment.

7.2.2 One-Dimensional, Partially Elastic Collision

The setup of this experiment is identical to the one above except for the masses of the cars and the setting of the timers as described below.

1. Add 200 g to the mass of the first car and weigh the total mass of the car; separately, weigh the total mass of the second car. Be sure to label the masses appropriately when recording them.

2. Position the cars as seen in Figure (7.1).

3. Set the second photogate timer to record two time periods with the *memory function*. After the collision, the second car passes through the second photogate followed immediately by the first car. The times of both are needed to calculate the two final velocities.

4. Gently, but firmly, push the first car toward the second car.

5. Make sure that both cars pass through the second photogate. Catch the second car after it passed through the photogate so that it does not bounce back!

6. Record the times for each car. The reading from the first photogate is the initial time of the first car. The reading from the second photogate is the final time of the second car (recall that the second car was initially at rest). **The memory reading is the total time of both cars.** To get the final time of the first car, take the memory reading (i.e., total time) and subtract the final time of the second car.

7.2.3 One-Dimensional, Totally Inelastic Collision

In this experiment, the setup is the same as for the first one except for the bumpers and sails as described below.

1. Place a sail on the top of the first car only since the cars stick together after the collision. Because the second car does not have a sail, add an extra accessory to it to equalize the masses. If you have not done so yet, remove the additional 200 g that was added in the previous setup.
2. Insert the needle and wax bumpers on the two facing sides of the cars. These replace the rubber band bumpers of the previous experiments.
3. Position the cars the same as seen in Figure (7.2).
4. Set the photogate timers to GATE mode.
5. Gently give the first car a push toward the second car.
6. Record the times in the appropriate section in the raw data sheet.

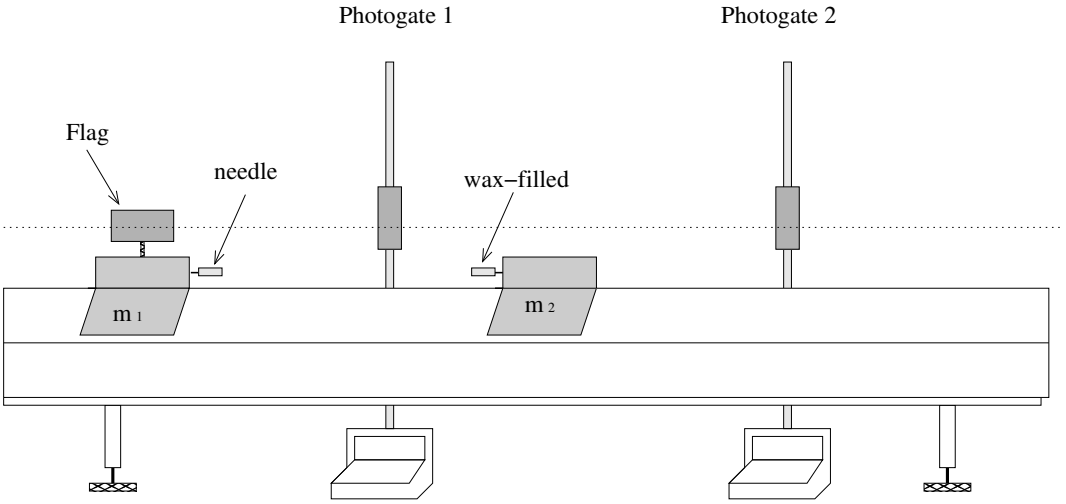

Figure 7.2: Schematic showing the experimental setup for the 1-D totally, inelastic collision.

7.2.4 Two-Dimensional Elastic Collision

This experiment uses the air table and two steel pucks of equal mass. The setup of the equipment is similar to the **Binary Systems** lab.

1. Position the two pucks in the corners nearest to you on the air table. Be sure that a fresh sheet of spark paper covers the air table surface.
2. Record the spark frequency of 20 Hz in your raw data sheet.
3. Power the air pump and practice launching the pucks such that they collide in the center of the table.
4. Launch the pucks as practiced and activate the spark timer at the same time by stepping on the floor pedal.
5. Step off the floor pedal just after the pucks hit the table's edge. Check with your TA to get approval and then make to note from which side the pucks were launched.
6. Appropriately label the beginning of the puck paths on the spark paper. Label the paths as # 1 and # 2 with arrows indicating velocity vectors as seen in Figure (7.3)

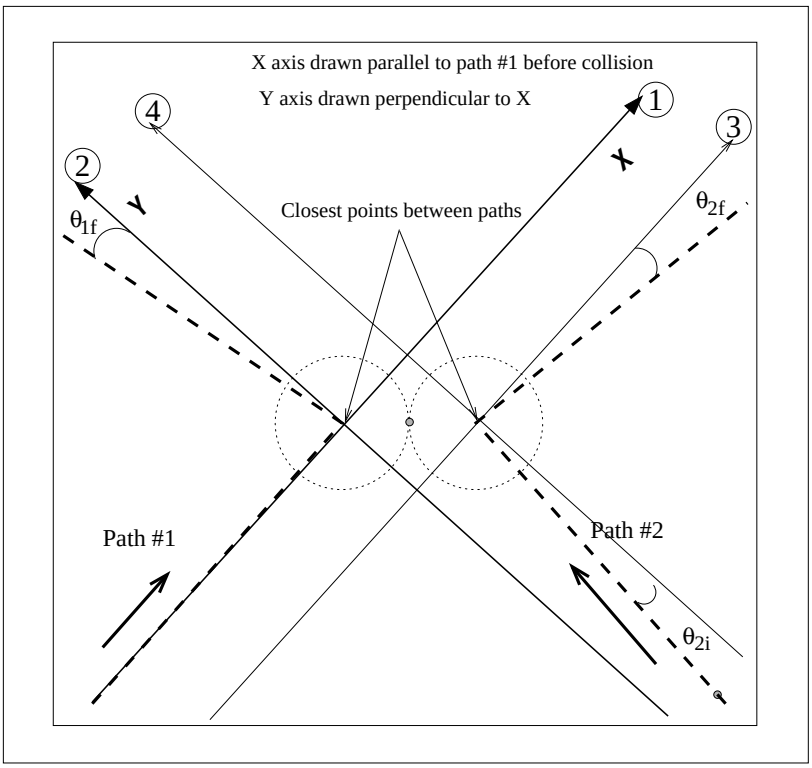

Figure 7.3: Diagram showing how to analyze data collected on the spark table. The darker dashed lines represent the carbon marks on the paper. The x-axis is drawn on top of track #1 (before the collision). The y-axis is perpendicular to the x-axis and drawn through the point where the change in direction occurs. The angles θ_{2i}, θ_{1f}, and θ_{2f} are measured and used to obtain the components of the velocities. Note the auxiliary axis drawn parallel to the main axis and required to measure θ_{2i}, and θ_{2f}. Circled numbers show the suggested sequence to trace the axis, using a straight edge –first the main axis, and then the auxiliary.

7. Use the trajectory of one of the initial paths to define the x-axis. You may want to draw your coordinate system on the paper. Use a ruler and a straight edge. If needed, ask your TA for guidance.

8. Measure and record the x and y components of the initial velocities for each puck near the collision point. That is, find $v_{1,ix}$, $v_{2,ix}$, $v_{1,iy}$, and $v_{1,iy}$. Do this by first measuring the initial velocity vectors for each puck, and then break them into their Cartesian components relative to your coordinate system. Be sure to measure the velocities as close to the collision point as possible to reduce errors.

9. Repeat the previous step for the final velocities, using the same coordinate orientation.

7.3 Analysis

i. For the experiment, One-Dimensional Perfectly-Elastic Collision, calculate the initial velocity of the first (v_{1i}) car and the final velocity of the second car (v_{2f}) using the time data collected, and the measured length of the sail. Take v_{1i} as the reference velocity and compare it to v_{2f} by relative percentage difference.

ii. For the experiment, One-Dimensional Partially-Elastic Collision, calculate each side of equation (7.2) to find the initial and final momenta; you may end up with unequal momenta. Take the initial momentum as the reference value and calculate the relative percentage difference between the initial and final momenta.

Calculate the energy loss Q from equation (7.4). Calculate the fraction of this loss to the initial kinetic energy; that is, Q/K_i.

iii. For the experiment, One-Dimensional Totally-Inelastic Collision, calculate the two velocities from the recorded time data and the length of the sail. Conservation of momentum predicts that the final velocity should be half the initial for two cars of equal mass in a totally inelastic collision. How close was your measured velocity to the prediction (use percentage of the relative error to quantify your answer)?

Calculate the fraction of kinetic energy lost – Q/K_i. Show **mathematically** that this quantity, Q/K_i, will equal $\frac{1}{2}$ for the parameters of this experiment (i.e do not use numerical values). Compare your experimental value to the accepted value by the percentage of relative error.

iv. For the experiment, Two-Dimensional Elastic Collision, use equations (7.10) and (7.11) to calculate the initial and final momenta separately for the x and y components. **For each component**, compare the initial momentum with the final momentum by relative percentage difference (taking the initial value as the reference value).

Lab 8

Inelastic Collisions

8.1 Background

In this lab, we investigate the conservation of momentum in two kinds of inelastic collisions – the ballistic pendulum and the angular collision. The ballistic pendulum is a classic physics experiment that is used to measure the velocity of projectiles by measuring the amount a pendulum moves when capturing a projectile. The angular collision involves dropping a steel hoop onto a spinning disk to slow it down.

When two objects collide and stick together, we call the collision totally inelastic. Although momentum is conserved in such a collision, Kinetic energy is not. A significant fraction of the initial Kinetic energy is lost to heat and vibration (internal energy). The following experiments demonstrate this.

8.1.1 Ballistic Pendulum

In this experiment, a pendulum of mass m_p, initially at rest, catches a steel ball with mass m_b and velocity v_b. The pendulum swings upward and pushes an indicator that measures the maximum angle θ reached.

By conservation of momentum, i.e. $\vec{p} = m\vec{v}$, the momentum of the system is the same before and after the collision

$$\vec{p_i} = \vec{p_f}$$
$$m_b v_b + m_p v = (m_b + m_p) v_p$$

where v_p is the velocity of the pendulum-ball system. Note that the pendulum is initially at rest, hence v is 0. Therefore,

$$m_b v_b + m_p \cdot 0 = (m_b + m_p) v_p$$
$$m_b v_b = (m_b + m_p) v_p \tag{8.1}$$

Note that this *inelastic collision* is one dimensional.

Although Kinetic energy is not conserved *during* the collision, it is conserved after the collision so that the gravitational potential energy at the maximum height of the pendulum's swing will equal the ball-pendulum kinetic energy just after the collision

$$\frac{1}{2}(m_b + m_p) v_p^2 = (m_b + m_p) g h \tag{8.2}$$

where h is the maximum vertical displacement of the *center of mass* of the pendulum with its captured ball. See Figure (8.1).

Solving equations (8.1) and (8.2) for v_p, we can set them equal to each other, and solve for the velocity of the ball, v_b. That yields

$$v_b = \frac{m_b + m_p}{m_b} \sqrt{2gh} \tag{8.3}$$

Letting $r_{\rm cm}$ be the distance from the pendulum's pivot point to its center of mass, as seen in Figure (8.1), we find that the maximum vertical displacement is

$$h = r_{\rm cm} - r_{\rm cm}\cos(\theta) = r_{\rm cm}[1 - \cos(\theta)]$$

Substituting this into equation (8.3) for the ball's velocity, we obtain

$$v_b = \frac{m_b + m_p}{m_b}\sqrt{2gr_{\rm cm}[1 - \cos(\theta)]} \tag{8.4}$$

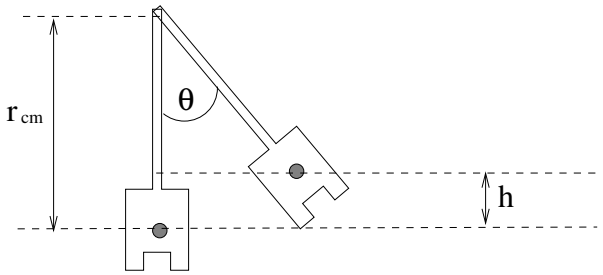

Figure 8.1: Schematic showing the height displacement, h, of the pendulum *after* the ball collided with it.

8.1.2 Angular Collision

In this experiment, a disk with moment of inertia, I_d, spins with an initial angular velocity, ω_i. A stationary steel hoop with moment of inertia, I_h, is dropped onto the disk. Due to the inelastic collision, the disk and the hoop will spin together at a slower final angular velocity, ω_f. We will measure these velocities using a tachometer.

Applying conservation of angular momentum, we can determine the final angular velocity, ω_f

$$I_d\omega_i + I_h\omega = (I_d + I_h)\omega_f$$

But as stated, the hoop is initially stationary, hence ω is 0:

$$I_d\omega_i + I_h \cdot 0 = (I_d + I_h)\omega_f$$

$$\omega_f = \frac{I_d}{I_d + I_h}\omega_i \tag{8.5}$$

where $\omega = 2\pi f$ and f is the frequency in revolutions per second.

The moment of inertia for the disk is

$$I_d = \frac{1}{2}m_d r_d^2 \tag{8.6}$$

where r_d is the radius of the disk.

The moment of inertia for the hoop is

$$I_h = \frac{1}{2}m_h(r_{\rm h,inner}^2 + r_{\rm h,outer}^2) \tag{8.7}$$

where $r_{\rm h,inner}$ and $r_{\rm h,outer}$ are the inner and outer radii of the hoop, respectively.

In the **Analysis Section**, we test the conservation of angular momentum by comparing the measured value of ω_f with the theoretical value calculated from equation (8.5).

8.2 Procedure

The following two experiments may be completed in any order. **Record all your measurements in SI units.**

8.2.1 Ballistic Pendulum

1. Remove the pendulum from the apparatus. Do this by unscrewing the pivot rod.
2. Weigh and record the mass of the pendulum, m_p, and the mass of the steel ball, m_b. Be sure that the two gold colored weights are securely attached to the pendulum.
3. Place the steel ball into the pendulum and find the position of the center of mass. One way to do this is as follows (see Figure (8.2)):
   (a) Tape a small piece of tape on the black part of the pendulum.
   (b) Balance the pendulum on a tuning fork or some other narrow flat object.
   (c) Slide the pendulum on the fork until it just begins to tip.
   (d) Mark the piece of paper on the pendulum right at this tipping point.
   (e) Slide the pendulum in the opposite direction until it just barely tips and, again, mark the piece of paper at that location.
   (f) Mark the midpoint of the two previous marks; this is the center of mass of the pendulum.

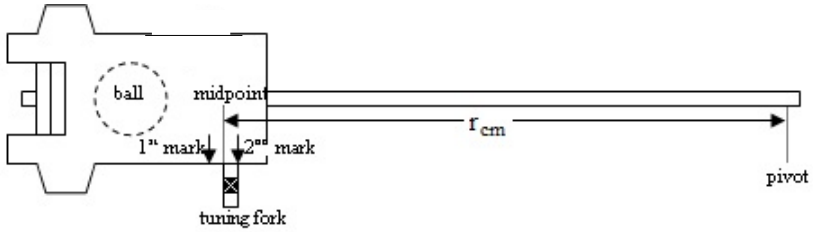

Figure 8.2: Schematic demonstrating how to determine r_{cm} of the pendulum.

4. Measure the distance from the pendulum's center of mass to its pivot hole. Record this value as r_{cm}.
5. Remove the ball from the pendulum using the silver latch, and reassemble the apparatus.
6. Place the ball in the launcher and set to the SHORT range. Be sure that the ball does not roll away from the launcher spring; slightly tilt the launcher to keep the ball in place using several sheets of paper.
7. Set the angle indicator on the pendulum apparatus to 0°. If you can't set it to 0°, make a note of what your initial angle reading is.
8. Launch the steel ball for a preliminary trial to get an idea what the final angle reading on the indicator should be.
9. Reset the launcher, fire the steel ball, and record the final angle in each of three trials. Be sure to label your data appropriately for each trial and subtract the initial angle reading if it is not at 0°.
10. Repeat steps 7 – 9 for the other two range settings (i.e. MEDIUM and LONG) of the launcher. Again, do three trials.

8.2.2 Angular Collision

1. Measure and record the mass of the disk, m_d, and the mass of the hoop, m_h. Here you will need to use the additional hanging mass provided. Ask your TA, if needed.
2. Measure and record the radius of the disk, r_d, the inner radius of the hoop, $r_{\mathrm{h,inner}}$, and the outer radius of the hoop, $r_{\mathrm{h,outer}}$.
3. We will use the tachometer to measure the rotation frequency.
   Note: There are 10 stripes on the disk. Since the tachometer reads each stripe, the final reading from the tachometer needs to be divided by 10 to give the correct value. This is the rotation frequency f in revolutions per minute (RPM).
4. Divide the responsibilities among your group according to: one person operates the tachometer near the outer edge of the disk, one person holds the hoop close to the disk in preparation to drop it into the slot, and one person records the readings.
5. Spin the disk and record the tachometer's displayed value for the initial rotation rate. **Do not use the very first reading displayed on the tachometer!**
6. Now drop the hoop into the slot on the spinning disk. Wait for a second to let the system stabilize before taking data. Use the tachometer to record the final frequency of the system.
7. Repeat previous steps for two more trials, each with a different initial rotation rate for the disk.

8.3 Analysis

8.3.1 Ballistic Pendulum

i. For each range setting of the launcher, calculate the average of the three angles measured. Now use the average angle for each range setting in equation (8.4) to calculate the ball's velocity, v_b. Clearly label the velocity with its corresponding range.

ii. Find the fraction R_{lost} of the initial kinetic energy that is lost to heat and vibration for each range setting. Use:

$$R_{\mathrm{lost}} = 1 - \frac{2(m_b + m_p)g r_{\mathrm{cm}}[1 - \cos(\theta)]}{m_b v_b^2} \tag{8.8}$$

which is obtained from the expression

$$R_{\mathrm{lost}} = \frac{K_i - V_{\mathrm{max}}}{K_i}$$

where K_i is the initial kinetic energy and V_{max} is the maximum gravitational potential energy.

iii. The fraction R_{lost} of initial energy lost is easily predicted by theory and depends only on the masses of the ball and pendulum. Derive this theoretical expression for R_{lost}. **Hint:** substitute v_b^2 from equation (8.4) into the equation (8.8).

iv. Calculate the theoretical value of R_{lost}; compare it to each of the experimental values calculated before; use relative error for comparison and discuss your results.

8.3.2 Angular Collision

i. To convert the earlier obtained rotation frequency f to angular velocity ω use

$$\omega = \frac{2\pi f}{60}$$

For each trial, calculate the initial and final angular velocities of the disk ω_i and ω_f, respectively. Note that the units of ω are radians per seconds.

ii. Use equations (8.6) and (8.7) to calculate the moment of inertia of the disk, I_d, and the moment of inertia of the hoop, I_h, respectively.

iii. Now evaluate equation (8.5) to find the theoretical value of the final angular velocity, ω_f, by using your calculated values of the moments of inertia and the initial angular velocity ω_i.

iv. Use relative error to compare the theoretical and experimental values of the final angular velocity. Explain any discrepancies.

v. The fraction R_{lost} of the initial rotational energy lost to heat and vibration is found similarly to that for the ballistic pendulum. The initial and final rotational energies are, respectively,

$$K_{\text{rotation,i}} = \frac{1}{2} I_d \omega_i^2$$

and

$$K_{\text{rotation,f}} = \frac{1}{2}(I_d + I_h)\omega_f^2$$

Hence, we obtain

$$R_{\text{lost}} = 1 - \frac{I_d + I_h}{I_d}\left(\frac{\omega_f}{\omega_i}\right)^2 \tag{8.9}$$

Calculate R_{lost} for each trial using equation (8.9).

vi. As was the case for the ballistic pendulum, R_{lost} is easily predicted from theory. In the case of the angular collision part of the lab, it depends only on the two moments of inertia. Derive this expression step-by-step. Show your work. **Hint:** Solve equation (8.5) for $\frac{\omega_f}{\omega_i}$ and substitute it into equation (8.9).

vii. Calculate the theoretical value of R_{lost} and compare it to each of the experimental values calculated before. Use relative error for comparison.

Lab 9

Torque and Moment of Inertia

9.1 Background

In this lab, the relationship between torque and angular acceleration is used to determine the moment of inertia of a disklike object with respect to its center of mass

$$I_{cm} = k\,MR^2, \tag{9.1}$$

where R and M are, respectively, the radius and mass of the object. In particular, we want to determine the value of k.

When the torque and the angular acceleration point in the same direction, their magnitudes are proportional to each other. That is

$$\tau = I\,\alpha, \tag{9.2}$$

where the moment of inertia I is the constant of proportionality. Hence to determine the moment of inertia I_{cm} we could apply a known torque $\vec{\tau}$ to the disk and measure the angular acceleration $\vec{\alpha}$. In general, the relationship between torque and the force producing the rotational motion is a vector relationship given by

$$\vec{\tau} = \vec{r} \times \vec{F}, \tag{9.3}$$

where the symbol $\times$ indicates a cross product; $\vec{r}$ is a vector from the axis of rotation to the point where the force $\vec{F}$ is applied; in general $\vec{r}$ and $\vec{F}$ are not perpendicular. The component of $\vec{r}$ perpendicular to $\vec{F}$ is called the lever arm $r_\perp = r\sin(\theta)$. Therefore by controlling the magnitude of the force and the lever arm, we can generate torques with prescribed values.

Equation (9.2) indicates that when the moment of inertia and the torque do not change in time, the torque produces a constant angular acceleration, α. In this case, we can obtain α from

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0), \tag{9.4}$$

where ω_0 and ω are the initial and final angular velocities; likewise θ_0 and θ are the initial and final angular displacements.

9.2 Procedure

1. Obtain a mass set.
2. Measure the diameter of the disk.
3. Use the vernier caliper to measure the diameter of the "silver hub"; Do check with your TA if you don't remember how to read the caliper.

4. Prepare your setup. Use Figure (9.1) for guidance and check with your TA before moving on.
5. Use the peg on the "silver hub" to wrap the string around it **4** times.
6. Hold the string in place so that it does not move when you attach the mass hanger, which weighs 50 g.
7. Ensure that the hanging mass will not hit the countertop while it is falling! Now release the string.
8. After the string completely left the disk, use the tachometer to measure the stripe frequency. Record the value in your raw data sheet and note that this reading is for 4 stripes. **Do not use the very first reading displayed on the tachometer!**
9. Keeping the same mass, repeat twice more the measurement of the stripe frequency; record your measurements.
10. Add mass to the hanger in increments of 50 g until 400 g (or the amounts given by your instructor). Measure and record three stripe frequencies for each new mass, as before.

9.3 Analysis

i. Use the stripe frequency to determine the spin frequency of the disk.

ii. For each trial (mass) average the values of the spin frequency of the disk.

iii. From the average spin frequency determine the angular velocity in radians per second.

iv. Use equation (9.4) to calculate the angular acceleration, α.

v. Determine τ using equation (9.3).

vi. Make a plot of torque v.s angular acceleration with torque on the vertical axis (even when it is the independent variable).

vii. Use a ruler to find the best fit line for your data and determine its slope. The slope is an estimate for the moment of inertia.

viii. Use your estimate for the moment of inertia and equation (9.1) to determine the experimental value of k for the disk.

ix. Compare by relative error this experimental k value to the accepted value of 0.5. Comment on the result.

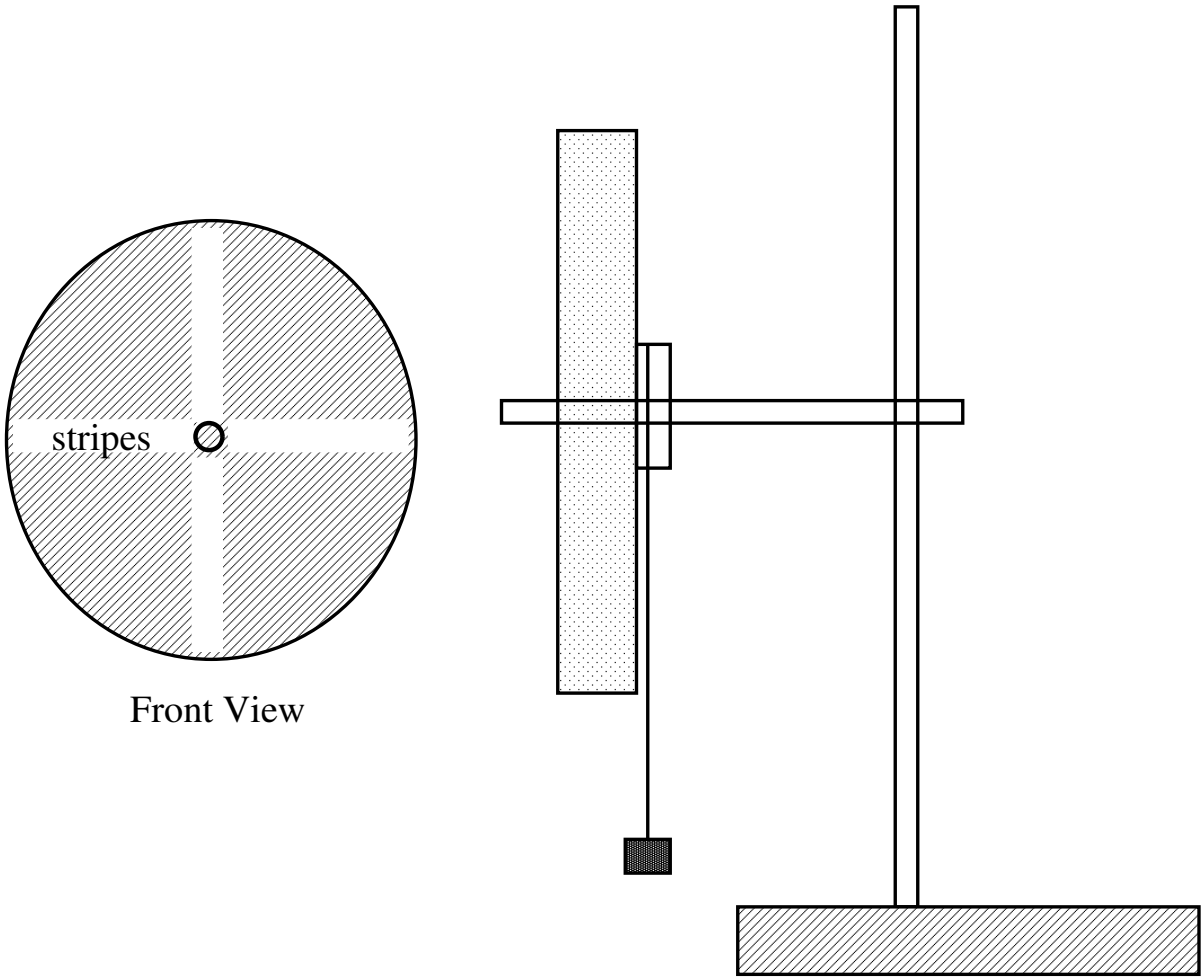

Figure 9.1: Schematic of the setup to determine the experimental value of k. The front view shows the stripes used by the tachometer to sense the motion of the disk. The tachometer will show the stripe frequency, which is four times the spin frequency of the disk.

Lab 10

Rolling Without Slipping

10.1 Background

When a cylindrical or spherical object rolls on a surface without slipping, the subsequent motion is a combination of translational and rotational motion. Slipping occurs when the tangential component of the gravitational force at the pivot point (i.e., where the object is in contact with the surface) exceeds the static friction force. As long as there is no slipping, energy losses to this form of friction are small, and the motion can be described by either a torque/acceleration approach or by a conservation of energy approach.

In this lab, we use both, the torque and energy, approaches, to evaluate the moment of inertia of the center of mass, $I_{\rm cm}$, which is proportional to the product of the mass and the square radius of the objects considered here. In particular, we estimate the constant of proportionality for various objects and compare our experimental values to their accepted values, which are listed in Table 10.1.

10.1.1 Torque/Acceleration Approach: Object Rolling on its Outside Edge

This approach is more involved than that of energy conservation but reveals more about the motion. Figure (10.1) shows an object of radius R with a center-of-mass moment of inertia $I_{\rm cm}$ on a ramp that is inclined at an angle θ with respect to the horizontal. The force of gravity acts at the object's center of mass to produce a torque τ about the pivot point P which causes a clockwise rotation and motion in the +x-direction down the ramp. This object is rolling on its outer edge so that the magnitudes of the translational velocity and acceleration are related to the magnitudes of the angular velocity and acceleration by:

$$v_{\rm cm} = \omega R \tag{10.1}$$

$$a_{\rm cm} = \alpha R \tag{10.2}$$

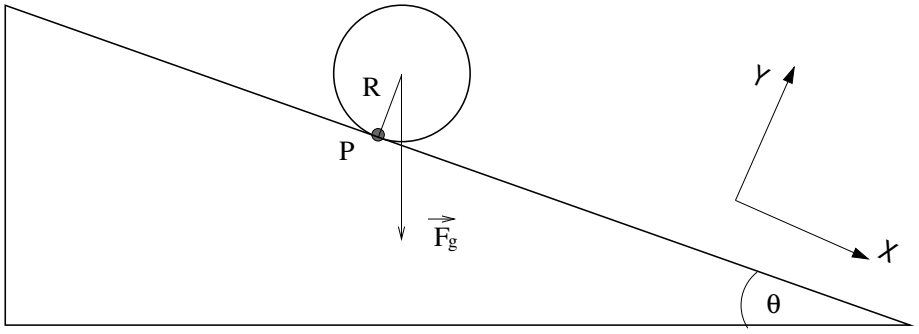

Figure 10.1: Schematic showing an object rolling down an inclined ramp. The radius of the object is R, while P is the contact point.

The moment of inertia about the pivot point is given by

$$I = mR^2 + I_{\text{cm}} \tag{10.3}$$

which comes from the Parallel Axis Theorem. We can express the moment of inertia about the center of mass as

$$I_{\text{cm}} = kmR^2 \tag{10.4}$$

where k is a dimensionless constant that depends on the geometry and the mass distribution of the object, and always has a range of $0 < k \leq 1$. Table 10.1 shows the accepted values of k for the objects considered in this lab. Using equation (10.4) in equation (10.3) gives

$$I = mR^2(1 + k) \tag{10.5}$$

The net torque can be written as

$$I\alpha = |\vec{R} \times \vec{F}|$$

where $\times$ represents the cross product. A cross product is defined by the formula $\vec{a} \times \vec{b} = ab\sin(\theta)$, where a and b represent the magnitudes of the vectors $\vec{a}$ and $\vec{b}$, respectively. Using Newton's second law for $|\vec{F}_g|$, we can write the previous expression as

$$I\alpha = mgR\sin(\theta) \tag{10.6}$$

Noting that the angular acceleration, α, of the center of mass about P is the same as that of P about the center of mass, we can write

$$\alpha = \frac{a_{\text{cm}}}{R} \tag{10.7}$$

Now we can use this expression along with equation (10.5) to obtain

$$mR^2(1 + k)\frac{a_{\text{cm}}}{R} = mgR\sin(\theta) \tag{10.8}$$

$$k = \frac{g\sin(\theta)}{a_{\text{cm}}} - 1 \tag{10.9}$$

Note: If the object was sliding down without rolling on a frictionless ramp, then it would resemble a block instead. That implies that $a_{\text{cm}} = g\sin(\theta)$ which gives $k = 0$ as expected in this situation.

10.1.2 Energy Conservation: Object Rolling on its Outside Edge

We note that the translational and rotational kinetic energies of the object are derived from the potential energy change as the object rolls down the ramp. Thus, when the object has dropped a vertical distance, h, the total energy is given by

$$mgh = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2 \tag{10.10}$$

For an object rolling on its outer edge, we use the definition of the rotational velocity,

$$\omega = \frac{v_{\text{cm}}}{R}, \tag{10.11}$$

and the expression for the moment of inertia about the center of mass to reduce and simplify equation (10.10) to

$$k = \frac{2gh}{v_{\text{cm}}^2} - 1 \tag{10.12}$$

10.1.3 Kinematics of the center of mass

From the previous analysis, it is clear that we need the acceleration of the center of mass and the final translation velocity of the rolling object along the ramp. We calculate them from the total time that the object is rolling down the ramp.

Since we only need acceleration and velocity along the ramp, this problem is one dimensional. Hence, from the kinematic equation for constant acceleration, we know that

$$s = s_0 + v_0 t + \frac{1}{2} a t^2, \tag{10.13}$$

where s is the distance travel, v_0 the initial velocity, a the acceleration, and t is the time elapsed. Applying this equation to the center of mass of the rolling object that starts from rest and travels a distance s in the time t, we have

$$s = \frac{1}{2} a_{cm} t^2. \tag{10.14}$$

Therefore, the acceleration of the center of mass is

$$a_{cm} = 2s/t^2. \tag{10.15}$$

Given the acceleration and the distance traveled, we can also use the equation

$$v_f^2 = v_0^2 + 2as \tag{10.16}$$

to obtain the final velocity of the center of mass. Since in our case $v_0 = 0$, we have

$$v_{cm}^2 = 2a_{cm}s. \tag{10.17}$$

Object	k
metal hoop	1.0
solid disk	0.50
solid sphere	0.40

Table 10.1: Accepted k values

10.2 Procedure

The event that we are going to time starts when the object is released at the top of the ramp and ends when it hits a cardboard box at the other end of the ramp.

1. Use masking tape to mark the starting point on the ramp.
2. For each object, measure the distance that the center of mass would travel on the ramp.
3. For each object, measure the height that the center of mass would have dropped by the end of the ramp.
4. Divide the following tasks within your team:
   - recording data,
   - positioning (and holding) the object at the agreed mark using a straight edge,
   - positioning the time-of-flight switch at the end of the ramp on the collision path of the object,
   - carefully positioning the photogate just in front of the object, so when the object is released, it breaks the light beam of the photogate triggering the timer.
5. Make a few practice trials until everyone in the team is comfortable with the timing of the event.
6. Run 3 trials for each object and record the times in your raw data sheet.

10.3 Analysis

First, determine the average time that took for each of the objects to roll down the ramp.

10.3.1 Torque approach

i. Use the average time and the distance travel on the ramp to determine the acceleration of the center of mass for each object.

ii. Use your a_{cm} values in equation (10.9) to find k for each object.

iii. Determine the percent difference between your calculated values and the accepted values.

10.3.2 Energy approach

i. Use the average time and the distance travel on the ramp to determine the final velocity of the center of mass for each object.

ii. Use height dropped, h, and final velocity, v_{cm}, in equation (10.12) to determine k for each object.

iii. Find the percent difference between your calculated values and the accepted ones.

Lab 11

Angular Momentum

11.1 Background

In this lab, we investigate the conservation of angular momentum in three applications. We measure a puck's velocity while it orbits on an air table; we see the effect of changing our moment of inertia while spinning on a rotating platform; and we see how changing the orientation of a bicycle wheel induces rotation on the same platform when initially still.

In previous labs, we studied the laws of conservation of momentum and energy. In this lab, we study another conservation law – conservation of angular momentum. Like linear momentum, angular momentum is conserved in isolated systems having no external interactions or forces.

We associate angular momentum with rotational motion and define it by

$$\vec{L} = \vec{r} \times \vec{p} \tag{11.1}$$

$$\vec{L} = \vec{r} \times m\vec{v} \tag{11.2}$$

$$\vec{L} = m(\vec{r} \times \vec{v}) \tag{11.3}$$

where $\vec{r}$ is the position vector, and $\vec{p}$ = m$\vec{v}$ is the linear momentum of the particle. The symbol $\times$ denotes the cross product of two vectors, which results in a vector perpendicular to the plane defined by the multiplying vectors; the orientation of the resultant vector is defined by the right hand rule. Therefore, $\vec{L}$ is a vector with a direction that is perpendicular to both $\vec{r}$ and $\vec{v}$, oriented along the axis of rotation – see Figure (11.1).

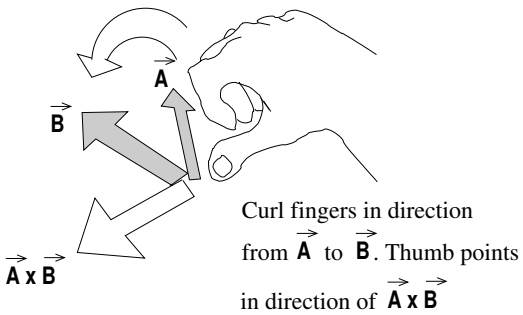

Figure 11.1: Schematic showing the right hand rule. Align your fingers with vector $\vec{A}$ and then curl them in the direction of vector $\vec{B}$. Your thumb points in the direction of the third vector (i.e. $\vec{A} \times \vec{B}$). Note that the direction of rotation is significant.

The magnitude of $\vec{L}$ is

$$|\vec{L}| = mrv_\perp \tag{11.4}$$

where $v_\perp$ is the component of the velocity perpendicular to $\vec{r}$.

The definition of $\vec{L}$ in the form given above is most readily applicable to particles or objects that can be treated as particles. However, for extended objects that are rotating about an axis that is within the object, our form of $\vec{L}$ applies to a mass element dm. Integrating over the object gives us

$$\vec{L} = I\vec{\omega} \tag{11.5}$$

where I is the moment of inertia, given by $I = kmR^2$, and $\vec{\omega}$ is the angular velocity. To find the direction of $\vec{\omega}$, we again apply the right hand rule: the fingers curl in the direction of rotation and the thumb points in the direction of $\vec{\omega}$. **Note that $\vec{L}$ and $\vec{\omega}$ point in the same direction!**

As with linear momentum, angular momentum is conserved at the component level. For example, if a system has $\vec{L}$ = [L_x L_y L_z] and an external force acts on it to change L_x only, then L_y and L_z remain unchanged – that is, they are conserved quantities of the system. We saw this for linear momentum in the projectile experiment, where v_x and p_x remained constant while v_y and p_y changed under the influence of the external force of gravity.

Also note that, as with linear momentum, conservation of angular momentum does not require conservation of energy, as we will see in the following experiments.

11.1.1 Orbiting Puck

This experiment shows that objects in eccentric or elliptical orbits (such as satellites and planets) conserve their angular momenta but not their kinetic energies. Similar to the **Binary Systems** lab, we approximate an orbiting object with a puck on an air table revolving around a central post to which it is attached by a spring.

To check for the conservation of angular momentum, we apply equation (11.4). An easy way to find $v_\perp$ is to notice that the puck moves in both the $\vec{r}$ and θ directions except when $\vec{r}$ is at its minimum or maximum. At these points, $\vec{r}$ does not change with time – that is, dr/dt = 0. Therefore, the r-component of velocity is zero and the only velocity is in θ, which is perpendicular to r –that is, $v_\perp$ = dθ/dt for those minimum and maximum points.

11.1.2 Changing of Moment of Inertia

This experiment demonstrates that the rate at which an object spins can be changed by adjusting the object's moment of inertia while in motion. The familiar example is when an ice skater or ballet dancer tucks in the arms while spinning in midair, which causes the skater or dancer to start spinning faster.

In midair, the system is isolated and angular momentum must be conserved. Tucking in the arms reduces the radius in the moment of inertia expression, given by $I = kmR^2$, which reduces the value of the moment of inertia. In equation (11.5), if the moment of inertia is reduced, and the value of angular momentum must stay constant, then the angular velocity $\vec{\omega}$ must increase to compensate.

In the experiment, we spin on a freely rotating chair, which is an isolated system for the z-components of angular velocity and angular momentum. These z-components, therefore, are conserved, and we can experience the same effects as skaters and dancers without any jumping.

11.1.3 Spinning Bicycle Wheel

In this experiment, we use a freely rotating platform and hold a spinning bicycle wheel in our hands. The wheel provides the angular momentum vector, which we reorient while standing on the platform to induce spinning in ourselves.

Changing the orientation of the wheel's angular momentum vector changes the magnitude of its z-component. Since

the z-component of total angular momentum is conserved for this system (consisting of the platform, wheel, and ourselves), changing its magnitude forces the system to compensate by spinning the platform. When we consider the quantities of angular momentum, we must keep in mind that the total angular momentum is the sum of the angular momenta associated with each moment of inertia. Although we have four different moments of inertia (one for the platform, one for the student on the platform, one for the wheel, and one for the wheel about the student), only two are relevant in our experiment, which are I_{wheel} and I_{student}. Therefore, we have the following conserved quantity in our system:

$$\vec{L}_{\text{z,total}} = \vec{L}_{\text{z,wheel}} + \vec{L}_{\text{z,student_on_platform}}, \tag{11.6}$$

where these values are taken as the magnitudes in the z-direction only.

As with the first experiment, kinetic energy is not conserved. That is, when we change the orientation of the wheel and induce spinning in the platform, the wheel maintains its initial angular speed. And as stated earlier $\vec{L}$ and $\vec{\omega}$ point in the same direction, which is true because I does not change while conducting the experimental steps.

11.2 Procedure

The following three experiments may be completed in any order. Only the first is quantitative; the other two are qualitative.

11.2.1 Orbiting Puck

1. Record the mass of the puck as 556 g.

2. Attach the puck to the central post on the air table with the provided lightweight spring. The mass of the spring is negligible and does not need to be measured (the spring constant also does not need to be measured). Be sure that the air table has a fresh sheet of spark paper covering it.

3. Place the second puck out of the way of the first but still in a corner of the table. This is needed to provide electrical continuity for the spark.

4. Practice launching the puck into a highly eccentric or elliptical orbit. **Make sure that the puck does not hit anything.**

5. Set the spark timer to 20 Hz and record this value in your raw data sheet.

6. Launch the puck as practiced and activate the spark timer for one full orbit plus a good extra half orbit. The additional portion from the second orbit should show evidence that the orbit is decaying. This is because the air table is not really frictionless.

7. Remove the spark paper and locate the two points where the radius is a maximum; connect them with a straight line, see Figure (11.2). You may also want to draw a vector indicating the direction in which the puck was orbiting.

8. Locate the two points where the radius is a minimum and connect them with a straight line. As Figure (11.2) indicates, the two axes should be perpendicular bisectors of each other.

9. Measure and record the values of $r_{\max}$ and $r_{\min}$.

10. Measure and record the distance between the two adjacent spark dots at the point of $r_{\max}$; repeat for $r_{\min}$.

    In the **Analysis Section**, these measured values are used to calculate the angular momentum at the maximum and minimum radii to verify the conservation law.

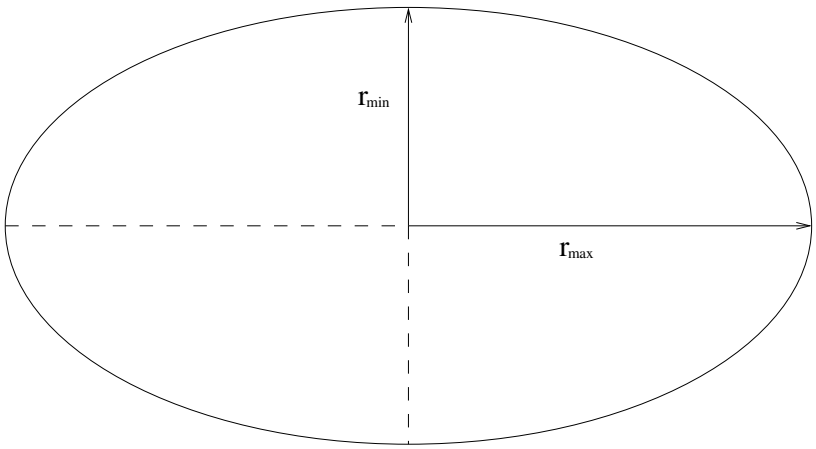

Figure 11.2: Schematic showing an elliptical orbit. The semi-minor and semi-major axes are $r_{\min}$ and $r_{\max}$, respectively (they are measured from the center to the edges).

11.2.2 Changing of Moment of Inertia

This experiment is qualitative, so approach it with a mind for describing your observations clearly.

1. Sit on the freely rotating table with your arms close against your sides. Be sure that your feet and knees are also close together. Note that you can use the provided hand weights, if you want to.
2. Have your lab partner spin you around at a moderate rate.
3. Extend your arms horizontally and record your observations.

11.2.3 Spinning Bicycle Wheel

This experiment is also qualitative, but keep in mind the magnitudes and orientations of the angular momentum vectors as these need to be sketched in the **Analysis Section**.

1. Spin the bicycle wheel as fast as possible.
2. Orient the angular velocity vector of the wheel, $\vec{\omega}_{\text{wheel}}$, towards your left on the horizontal plane. See Figure (11.3). Do this before stepping onto the platform.
3. Step onto the platform. Be sure not to change the orientation of $\vec{\omega}_{\text{wheel}}$ when doing this. After stepping on the platform, make sure you are not moving. If you are moving, have one of your team members use their foot to stop the platform from spinning.
4. From your *stationary position*, reorient $\vec{\omega}_{\text{wheel}}$ by 45° downward and record your observations.
5. Return to the starting conditions of step 2 above (as seen in Figure (11.3)). Step off the platform and re-spin the wheel.
6. Now reorient $\vec{\omega}_{\text{wheel}}$ by 45° upward and record your observations. between directly up and directly left.
7. Step off the platform, re-spin the wheel, and orient $\vec{\omega}_{\text{wheel}}$ to point directly up, see Figure (11.4).
8. Step back onto the platform. **Again, make sure you are completely stationary before continuing to the next step!**
9. Reorient $\vec{\omega}_{\text{wheel}}$ by 45° to the left and record your observations. Straighten the wheel so that $\vec{\omega}_{\text{wheel}}$ points directly up. Ensure that you are once again completely stationary before continuing. Now reorient $\vec{\omega}_{\text{wheel}}$ by 45° to the right; record your observations.

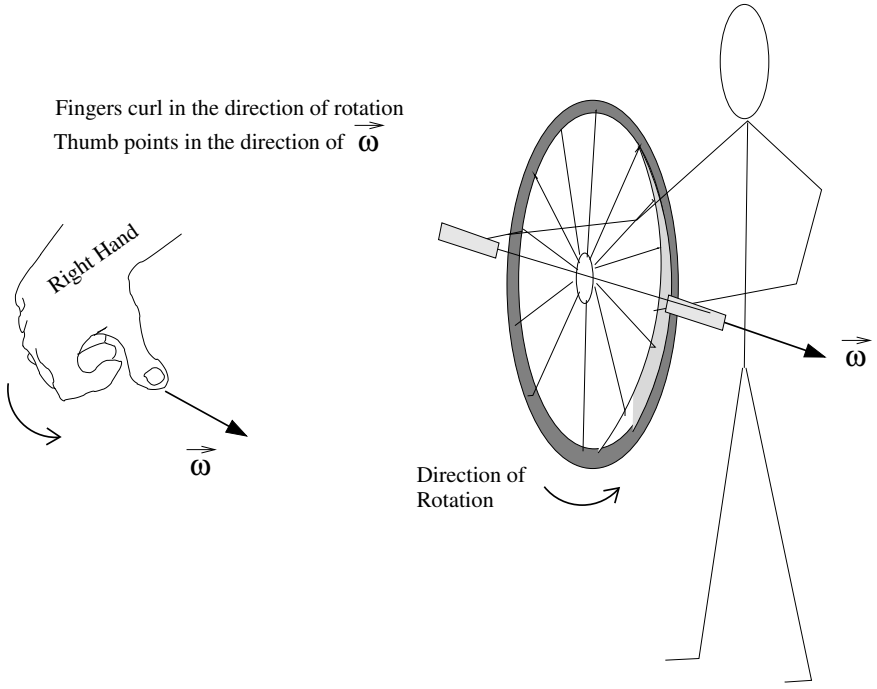

Figure 11.3: Schematic showing how to align the horizontal angular velocity vector of the wheel, $\vec{\omega}_{\mathrm{wheel}}$.

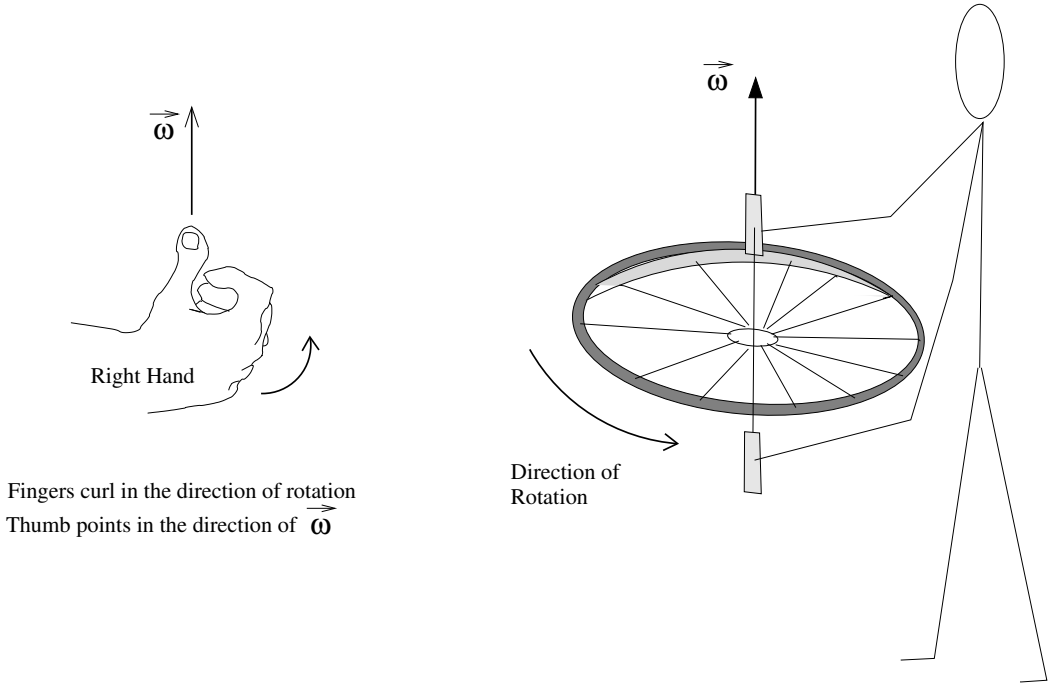

Figure 11.4: Schematic showing how to align the angular velocity vector of the wheel, $\vec{\omega}_{\mathrm{wheel}}$, upward.

10. Repeat the previous step for 45° to the front and then to the back. Again, make sure you are *stationary* before you reorient $\vec{\omega}_{\mathrm{wheel}}$ to the front and then the back. Record your observations for both alignments.

11. Return $\vec{\omega}_{\mathrm{wheel}}$ to its starting orientation of directly up (as seen in Figure (11.4)). Again, make sure you are

completely stationary while standing on the platform. Quickly stop the wheel's rotation by grabbing the edge of the wheel with your hand. Record your observations.

12. Return to the starting conditions of step 7 (i.e as seen in Figure (11.4)) and step back onto the platform. **However, do not spin the wheel as fast as you did before.** Be completely stationary on the platform before proceeding.

13. Flip the wheel and record your observations. That is, rotate the wheel axis by 180° so that $\vec{\omega}_{\text{wheel}}$ is pointing directly down.

11.3 Analysis

11.3.1 Orbiting Puck

i. For the points of $r_{\max}$ and $r_{\min}$, use your measured values of distance between the spark dots and the time interval to calculate $v_\perp$. That is for each location, determine $v_\perp$.

ii. Calculate $\vec{L}$ for each of the above points using equation (11.4).

The value of $\vec{L}$ at each extreme point should be the same, if angular momentum is conserved. Compare your calculated values with each other by relative percentage difference, taking the first value as the reference. Explain any discrepancies.

iii. What is the direction of the angular momentum vector $\vec{L}$ for the orbiting puck?

iv. If the air table has no friction, then the energy should be conserved. Yet the puck clearly has greater kinetic energy at $r_{\min}$ than at $r_{\max}$. So how is energy conserved? Explain.

11.3.2 Changing of Moment of Inertia

i. Suppose your initial moment of inertia and angular velocity are I_i and ω_i, respectively. If extending your arms doubles your moment of inertia so that $I_f = 2I_i$, then what is ω_f in terms of ω_i? Show your work.

ii. For the same conditions as above, calculate and compare the initial and final rotation energies using

$$K_{\text{rotation}} = \frac{1}{2} I\omega^2. \tag{11.7}$$

Why are they different? Explain.

11.3.3 Spinning Bicycle Wheel

i. When you stepped onto the platform in step 3 of the **Procedure Section**, the z-component of the system's angular momentum was zero. But when you reoriented the wheel in step 4, you attempted to change the magnitude of that z-component.
Show conservation of angular momentum by sketching the system's total angular momentum vectors and their z-components for both before and after the reorientation. What can you conclude about the magnitudes of the wheel's angular momentum and your angular momentum?

ii. In step 6 of the **Procedure Section**, you reoriented the wheel in the opposite direction of that in step 4. Again, show the conservation of angular momentum by sketching the vectors. Explain why you start rotating in the opposite direction of that in step 4.

iii. In step 9 of the **Procedure Section**, you reoriented the wheel to your left and then to your right when it was initially pointing directly up. As before, sketch the angular momentum vectors to show conservation.
Why does reorienting the wheel through each of the four different directions in steps 9 and 10 produce a result different from that in step 4?

iv. In step 11, you quickly stopped the spinning wheel. Sketch the angular momentum vectors and explain your subsequent motion.

v. If the **initial spin rate** of the wheel were the same in steps 9, 10, and 11, would your rotation rate (after completing the instructed action for each step) be the same in each? Explain.

vi. In step 13 of the **Procedure Section**, you flipped the wheel. Explain your subsequent motion by sketching the conserved angular momentum vectors.

vii. We stated in the **Background Section** that the angular speed of the wheel does not change when reoriented even though it induces rotation in you; kinetic energy is clearly not conserved. If the initial kinetic energy of the system was the rotational energy of the wheel, which remains constant, where did your rotational energy come from?

Lab 12

Harmonic Oscillators

12.1 Background

In this laboratory experiment we study the behavior of harmonic oscillators. Oscillators occur in a variety of forms and permeate the natural world. Man-made objects such as musical instruments, clocks, radios, television sets, and lasers make good use of oscillators. Clouds and other atmospheric phenomena exhibit strong vertical oscillatory behavior in the atmosphere. The repetitive elliptical or circular motion of orbiting astronomical objects or electrons in an atom is oscillatory in two spatial dimensions, as is the motion of rotating weather systems such as hurricanes and tornadoes. When oscillations are coupled between adjacent regions they can propagate through the region as a wave. Examples of this are water waves, sound waves, and electromagnetic wave phenomena such as radio waves or light. Oscillatory motion is obviously important to understand.

Harmonic oscillations are repetitive in nature, namely they repeat after some time interval T, called the *period* of the oscillation. For example, if an oscillation or rotation occurs at a frequency f = 20 cycles per second, the time for one oscillation is $T = 1/f = 1/20$ second. The period and frequency of an oscillation are thus simply related by

$$T = \frac{1}{f} \tag{12.1}$$

The units of frequency, f, are cycles per second, or Hertz (Hz). Any repetitive oscillation can be considered to be the sum of sinusoidal oscillations at the fundamental frequency $f_0 = 1/T$ as well as at multiples or "harmonics" of f_0. A sinusoidal oscillation has the form

$$x = x_0 \cos(\omega t) \tag{12.2}$$

where ω is the angular frequency of the oscillations in radians per second. During one cycle of an oscillation, the argument of the cosine function (ωt) changes by 2π radians. The angular frequency, ω is therefore 2π times the number of oscillations per second, or

$$\omega = 2\pi f = \frac{2\pi}{T}. \tag{12.3}$$

The above provides the fundamental relations we need to study oscillatory behavior.

12.1.1 The Mass-Spring Oscillator

The classic example of a harmonic oscillator is a mass-spring system. When the mass is displaced from its equilibrium position the spring exerts a restoring force that accelerates the mass back toward the equilibrium point. Upon reaching the equilibrium point, however, the mass has inertia or momentum and overshoots. This keeps happening, causing the mass to oscillate around the equilibrium position. The oscillations die out after some time as their energy is converted to heat by frictional forces.

If the restoring force on an object is proportional to the object's displacement from the equilibrium position, the equations governing the motion are relatively simple. This is the case for the mass-spring oscillator and is usually

the case for more complex systems when the displacement from equilibrium is small. For the mass-spring system the restoring force is given by $F = -k_s x$, where k_s is the spring constant and x is the displacement of the mass from its equilibrium position. This equation is called Hooke's Law. Applying Newton's second law, we have that

$$F = -k_s x = ma \tag{12.4}$$

Since acceleration $a = \frac{dv}{dt}$ and velocity $v = \frac{dx}{dt}$, we get

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Thus,

$$-k_s x = m\frac{d^2x}{dt^2} \tag{12.5}$$

This is a *differential equation* since it relates x to derivatives of x.

A common way of solving a differential equation is to make an educated guess about the answer and substitute the guess into the equation to see if it satisfies the equation. Using knowledge of derivatives, one can show that the solution

$$x = x_0 \cos(\omega_0 t) \tag{12.6}$$

satisfies equation (12.4) when the angular frequency is given by

$$\omega_0 = \sqrt{\frac{k_s}{m}} \tag{12.7}$$

The motion is therefore sinusoidal or oscillatory in nature, as expected. An important point to note is that the frequency is independent of the amplitude of the oscillation. This is why a piano string gives the same basic note whether it is struck softly or loudly.

12.1.2 Resonance

The natural frequency of an oscillator can also be measured by exciting or *driving* the oscillator at different frequencies and observing the response. When the excitation frequency is at or near the natural oscillation frequency, the amplitude of the oscillations will become large and the oscillator can go wild. This phenomenon is termed *resonance*. Just as on a playground swing, application of the exciting force at just the right time will cause the oscillations to increase in size. This is easy to demonstrate, for example, in **The Pendulum Section** of this lab.
In this lab, we will have the spring connect to a mechanical wave driver. A magnetic coil is driven by a variable frequency power source (called sine wave generator), causing a plunger to move up and down at the driving frequency. This couples energy into the oscillator through the spring, causing the system to oscillate at the same frequency as the driving signal. The oscillation amplitude is small when the driving frequency is different from the resonance frequency and large at resonance.

12.1.3 The Pendulum

The pendulum has historically been a good timing device by virtue of the fact that its oscillation period/frequency depends only on the length of the pendulum ℓ, and the acceleration of gravity g, which are well-defined quantities. The restoring force in a pendulum is given by $mg\sin(\theta)$, where θ is the angle from vertical. If θ is small, the small angle approximation gives $\sin(\theta) \simeq \theta$. Therefore the restoring force for small θ is

$$F_{\text{restoring}} = mg\sin(\theta) \simeq mg\theta \tag{12.8}$$

Considering the displacement to be the angle θ rather than the linear distance of the swing, the restoring force is proportional to the displacement. This gives rise to sinusoidal oscillations of the form

$$\theta = \theta_0 \cos(\omega_0 t), \tag{12.9}$$

where ω_0 is given by

$$\omega_0 = \sqrt{\frac{g}{\ell}} \tag{12.10}$$

and ℓ is the length of the pendulum. Note that equation (12.10) is independent of mass.

The pendulum can be made into a two-dimensional oscillator by launching it along a path that circles around the equilibrium point. This is called a conical pendulum. In the two-dimensional case, the period of the pendulum is given by

$$T = 2\pi\sqrt{\frac{\ell\cos(\theta)}{g}} \tag{12.11}$$

where θ is the angle that the pendulum is displaced from vertical, as seen in Figure (12.1).

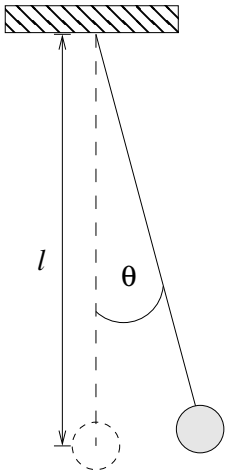

Figure 12.1: Schematic for a pendulum showing the angle θ, the length ℓ, and the equilibrium position, which is indicated by the dashed lines.

12.2 Procedure

For each type of oscillator below, we will measure the period or frequency of the oscillations and compare the result with the values predicted by theory.

12.2.1 Mass-Spring Oscillator

1. Determine the spring constant. Remove the spring from the *mechanical wave driver* and use the metal stand provided. Remember that the mass hanger itself has a mass of 50 g. Add known masses, increment by 50 g, to the mass hanger. Measure the change in the length of the spring, and fill in the table in the raw data sheet.

2. Now determine the frequency of a mass-spring oscillator by measuring its period over a number of oscillations using a stop watch. For this, hang a total mass of 150 g from the spring. This should make the measurement of the oscillations easier to time.

3. Gently pull down the mass, release it and start the stopwatch at the same time. Count the number of full oscillations and record them along with the time. Each group member should do at least one measurement!

4. Compute the period by dividing the time by the number of oscillations. This provides the time for one oscillation.

5. Average the values of the periods.

12.2.2 Resonance

1. Remove the spring from the stand and connect the upper part of the spring to the *mechanical wave driver*. This allows the plunger to move up and down at the driving frequency.
2. Make sure that the *mechanical wave driver* is in the "unlocked" position. For this, look for the silver lever on the side of the device and the writing on the underside of the device. See Figure (12.2) for the setup.

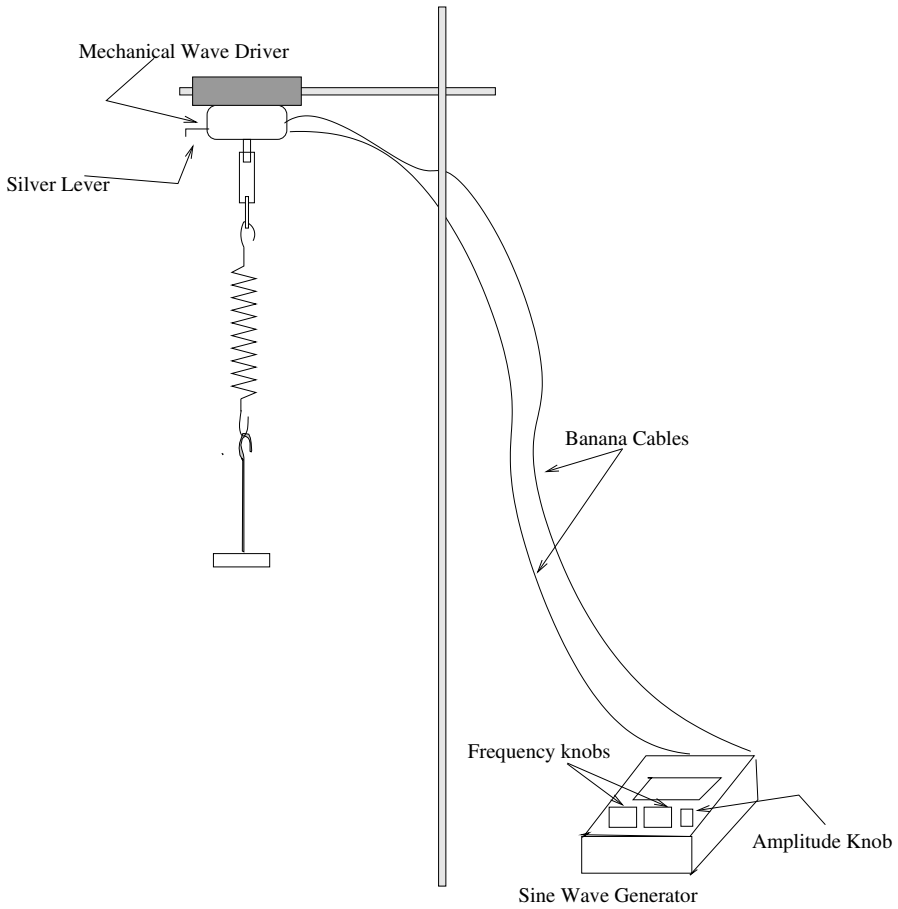

Figure 12.2: Schematic for the resonance setup. The silver lever is used to lock and unlock the driver.

3. Attach a total mass of 150 g to the spring.
4. Turn on the *sine wave generator* and dial to a starting frequency of 1 Hz.
5. Carefully adjust the driving frequency around the resonance point to determine the value at which resonance is the strongest. For that, use the middle dial on the *sine wave generator* to adjust the frequency by 0.1 Hz. Record this value as your resonance frequency, f_r. Keep the amplitude of the excitation small to avoid damaging the driving system.
6. Now vary the driving frequency from below to above resonance, observe the response and make a note in your raw data sheet.
7. Turn off the *sine wave generator*.

12.2.3 The Pendulum

1. Measure the length of the pendulum, ℓ. Do **not** step onto the countertop!

2. Now measure the period T of the pendulum using a photogate. Make sure that the photogate is set to the PEND mode with the time resolution set to 1.0 ms. In this mode the gate accurately determines one period of the pendulum.

3. Measure the period for several different initial amplitudes, such as 0.1 m to 0.9 m, incremented by 0.2 m.

4. The pendulum hanging straight down is called the equilibrium position. Use a meter stick to "pull" the pendulum straight back to the given amplitudes. Release the pendulum. *Be careful when doing this not to hit the photogate.*

5. For each amplitude, record the time from the photogate.

12.2.4 2-D Pendulum

1. First, remove everything around the pendulum including the photogate. Practice creating a circular orbit. This might take a few runs until you get a good one.

2. Now make a circle with a radius of approximately 0.8 m, which should work best. Record your actual radius in the raw data sheet.

3. Using a stopwatch, measure the time for one or more orbits of the pendulum. Record the number of orbits N and the time t from the stopwatch.

4. Obtain the period T by dividing the time by the number of orbits.

12.3 Analysis

12.3.1 Mass-Spring Oscillator

i. Show that $x = x_0\ \cos(\omega_0 t)$ is a solution of equation (12.5). Hint: substitute the solution in the equation.

ii. Compute the natural frequency f in Hz from the average period.

iii. Calculate the spring constant in SI units. Do this by creating a plot of mass and length, and obtain the spring constant as done in previous labs (e.g see **Binary System lab**). Make sure to include this plot, with correctly labeled axes, in your lab report.

iv. Use equations (12.3) and (12.7) to obtain the theoretical value of the frequency f.

v. Compared by relative percentage error the theoretical value of f to the value determined from your data. Discuss any differences.

12.3.2 Resonance

i. Using their relative percentage difference, compare the measured natural frequency f to the measured resonant frequency f_r. Take the natural frequency as the reference value. Why are they different? Explain.

12.3.3 The Pendulum

i. From equation (12.10) determine the theoretical value for the period.

ii. Compare, by relative percentage error, the measured periods with that predicted by theory.

iii. The slight change of the period with amplitude is caused by the restoring force becoming non-linear due to the $\sin(\theta)$ term in equation (12.8). Is that apparent in your data? Explain.

iv. Calculate the relative percentage difference in the period of the pendulum for large amplitude swings versus small amplitude swings. Take the small amplitude period as the reference value.

12.3.4 Conical Pendulum

i. First determine θ, which is needed for equation (12.11). This can be done by grabbing the pendulum and pulling it out to a certain amplitude and using this as the radius of the orbit. Recall that you measured the length of the string. These two values and some trigonometric relation allows you to determine θ. Hint: see Figure (12.1)).

ii. Now use equation (12.11) to calculate the theoretical value of the period.

iii. Compare your measured period to the theoretical value by relative error and explain any differences.

Part I

Graph Paper